Spatial econometrics (1)

Applied Econometrics for Spatial Economics

Hans Koster

Professor of Urban Economics and Real Estate

1. Introduction

- Hans Koster → URBANECONOMICS.NL
  - Professor of Urban Economics and Real Estate
  - h.koster@vu.nl
  - Lectures
  - Programme director of STREEM (streem.sbe@vu.nl)

- Canvas
  - Be aware of the recap materials under *modules*
  - If there is anything unclear, let me know!

1. Introduction

- **A few announcements on the programme**
  - **Enrol with the following link to the MSc STREEM announcement page https://canvas.vu.nl/enroll/FXW8BL**
  - **Thesis support sessions: October 4 and 11, 12:45-13:30**
  - **Drinks on October 28, 15:00, after exam**

1. Introduction

- This course
  - Learn about advanced tools and techniques important for
    - » Urban & Regional,
    - » Real Estate,
    - » Transport and
    - » Environmental Economics

- Do not hesitate to ask questions during the class!

- Notation on slides
  - Most important concept are underlined
  - Questions (via Menti), exercises and applications
    → On red slides

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1. Introduction

- **How to study:**
  → Attend offline lectures and tutorials
  → Work together on assignments on campus
  → Read and study syllabus

1. Introduction

- Tutorials
  - Thomas de Graaff (t.de.graaff@vu.nl, thomasdegraaff.nl)
  - For any questions on the assignments/
  - tutorials please ask Thomas

- Please subscribe to groups on Canvas *before Tuesday 23:59*
  - Groups of 3

1. Introduction

- This week
  - Learn about how to deal with spatial data
  - … and spatial econometrics

- Plan:

Lecture #1:	Spatial data
Lecture #2:	Spatial autocorrelation and regressions
Lecture #3:	Spatial regressions (cont'd)
Assignment:	Open space and school quality

- **Some remarks on matrix notation**
  - **Use bold symbols for vectors**

$$\boldsymbol{x} = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix}$$

  - **Use bold symbols and capitals for matrices**

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

  - **Identity matrix**

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow \boldsymbol{IX} = \boldsymbol{X}$

  - **Inverse $\boldsymbol{X}^{-1}$ is matrix equivalent of $1/x$**

$\rightarrow \boldsymbol{X}^{-1}\boldsymbol{X} = \boldsymbol{X}\boldsymbol{X}^{-1} = \boldsymbol{I}$

  - **More details in the appendix of the syllabus**

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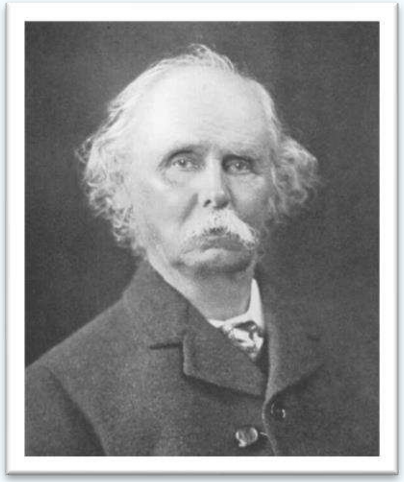
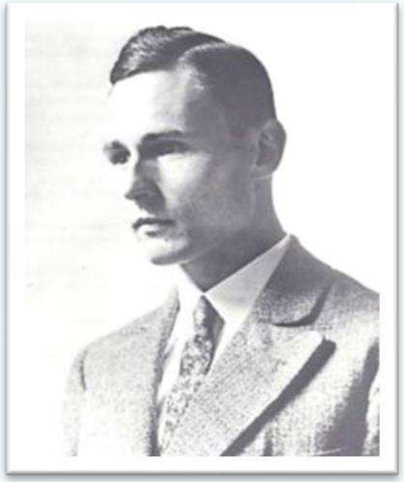
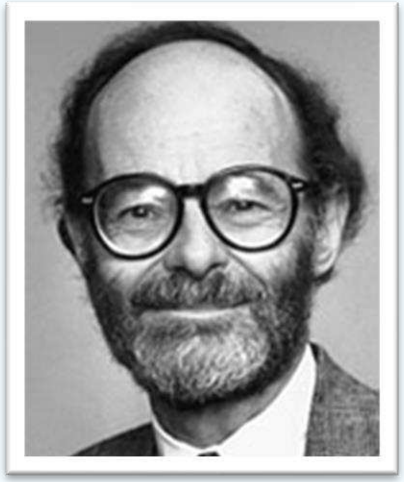
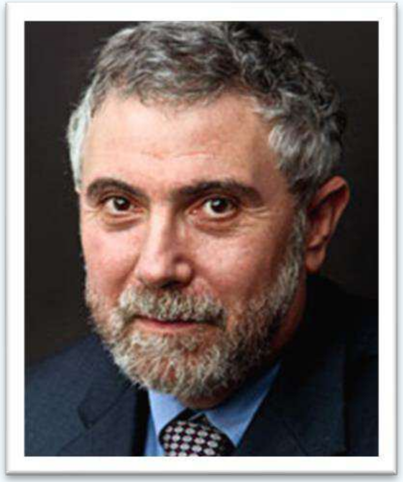
2. Space in economics

- Many economic processes are spatially correlated
  - Tobler's first law of geography
- Most economics models are "topologically invariant"
- New economic fields have emerged
  - Urban economics
  - New economic geography (NEG)
- Synergy with other fields
  - Economic geography
  - Regional science
  - GIS

- **Economists and space**


YOU?

- **Spatial econometrics**

- **40-50s – mainly domain of statisticians**

- **Cliff and Ord (1973): "Spatial autocorrelation"**

- **Paelinck and Klaassen (1979): "Spatial Econometrics"**

- **Rapid growth since Anselin (1988)**

- **New estimators, tests and interpretation**
  - ***e.g.*** **Kelejian and Prucha (1998, 1999, 2004, 2007, 2010)**

2. Space in economics

- **Spatial modelling is becoming increasingly important**
  - **New and geo-referenced data**
  - **Advanced software**
  - ***New methods and regression techniques!***

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3. Spatial data structure

- **Time is simple**
  - **Natural origin**
  - **No reciprocity**
  - **Unidirectional**

$$x_{t-3} \longrightarrow x_{t-2} \longrightarrow x_{t-1} \longrightarrow x_t$$

- **Linear space (*e.g. beach*) is different**
  - **No natural origin**
  - **Reciprocity**
  - **Unidirectional**

$$x_1 \longleftrightarrow x_2 \longleftrightarrow x_3 \longleftrightarrow x_4$$

3. Spatial data structure

- **Two-dimensional space becomes even more complex**
  - **No natural origin**
  - **Reciprocity**
  - **Multidirectional**

$$\begin{tikzcd}
& x_1 \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
x_3 \arrow[rr, leftrightarrow] & & x_2
\end{tikzcd}$$

- $i = 1,2,3$ **can refer to point data, areas, grids**

3. Spatial data structure

- First, we have to define the spatial structure of the data
- Specified through a spatial weights matrix
- Spatial weights matrix W:
  - Consists of $n \times n$ elements
  - Discrete or continuous elements
- How to define weights?
  - Euclidian distance
  - Network distance
  - Spatial interactions
  - Social networks

3. Spatial data structure

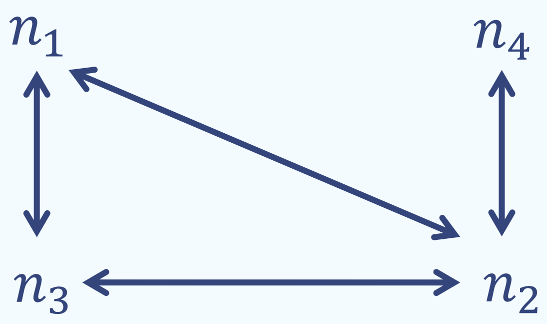
- **How to define spatial matrices?**

- **Contiguity matrix**
  - Adjacent → 1st order contiguous
  - Neighbours of neighbours → 2nd order contiguous

- **Distance matrix**
  - k-nearest neighbours
  - Inverse distance weights ($1/distance$)
  - Cut-off distance

3. Spatial data structure

- **Let's provide an example of a contiguity matrix**



to (columns), from (rows)

W	n_1	n_2	n_3	n_4
n_1	0	1	1	0
n_2	1	0	1	1
n_3	1	1	0	0
n_4	0	1	0	0

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- **Matrices can be standardised**
  - **Different principles can be used**
  - **Most common: *row-standardisation*:**

    $$w_{ij}^{*} = \frac{w_{ij}}{\sum_{k=1}^{n} w_{ik}}$$

    where k are other locations

- **Interpretation of**
  - $\sum_{j=1}^{n} w_{ij}$**: sum of connections to neighbours**
  - w_{ij}^{*} **denotes the share of connections to neighbours**

3. Spatial data structure

- Remarks regarding distance weight matrices
  - Check for exogeneity of matrix
  - Connectivity
  - Symmetry
  - Standardisation
  - Distance decay

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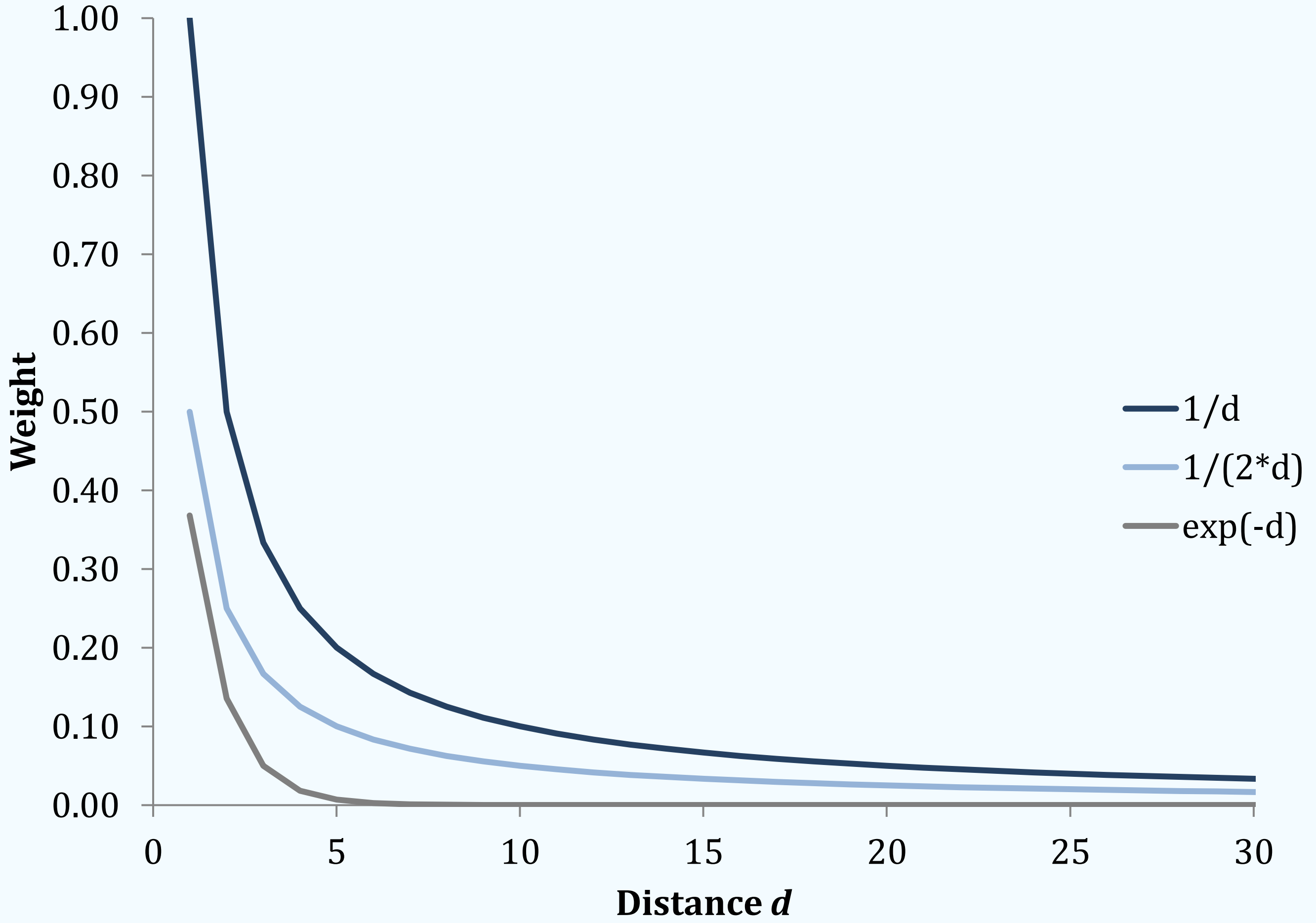
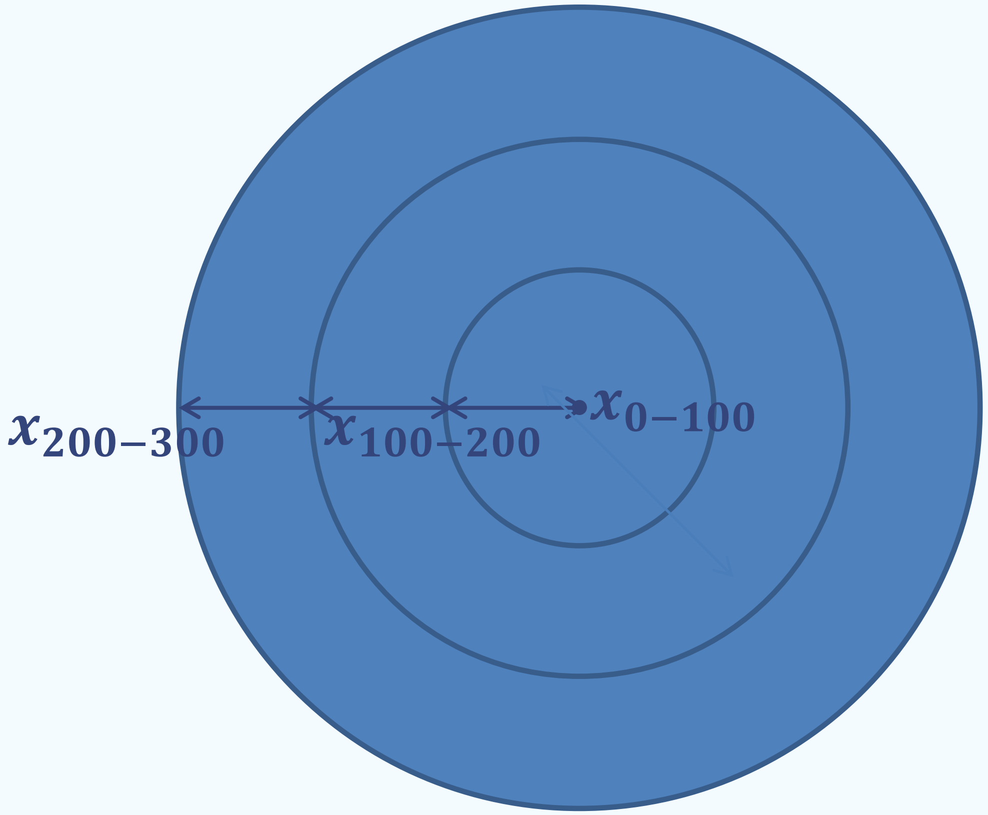
- **Choice of distance decay is arbitrary**
  - Sometimes theory may help
  - May also try to find the optimal decay parameter empirically



3. Spatial data structure

- Choice of distance decay is arbitrary
  - An alternative is to forget about specifying W
  - Alternatively, use different x-variables capturing concentric rings
  - Average of x-variable for different distance bands

- **Choice of distance decay is arbitrary**
  - **e.g.** $y = \alpha x_{0-100} + \beta x_{100-200} + \gamma x_{200-300} + \epsilon$



3. Spatial data structure

- **How to define spatial weight matrix using software**
  - SPATWMAT **in STATA, based on geographic coordinates**
  - SPWEIGHT **in STATA**
  - **Geoda**
  - SPATIAL STATISTICS TOOLBOX **in ArcGIS**
  - SPDEP **in *R***

- **Concentric rings should be calculated manually**

4. Modifiable areal unit problem

- Usually we do not have space-continuous data
  - ‘Dots’ to ‘boxes’

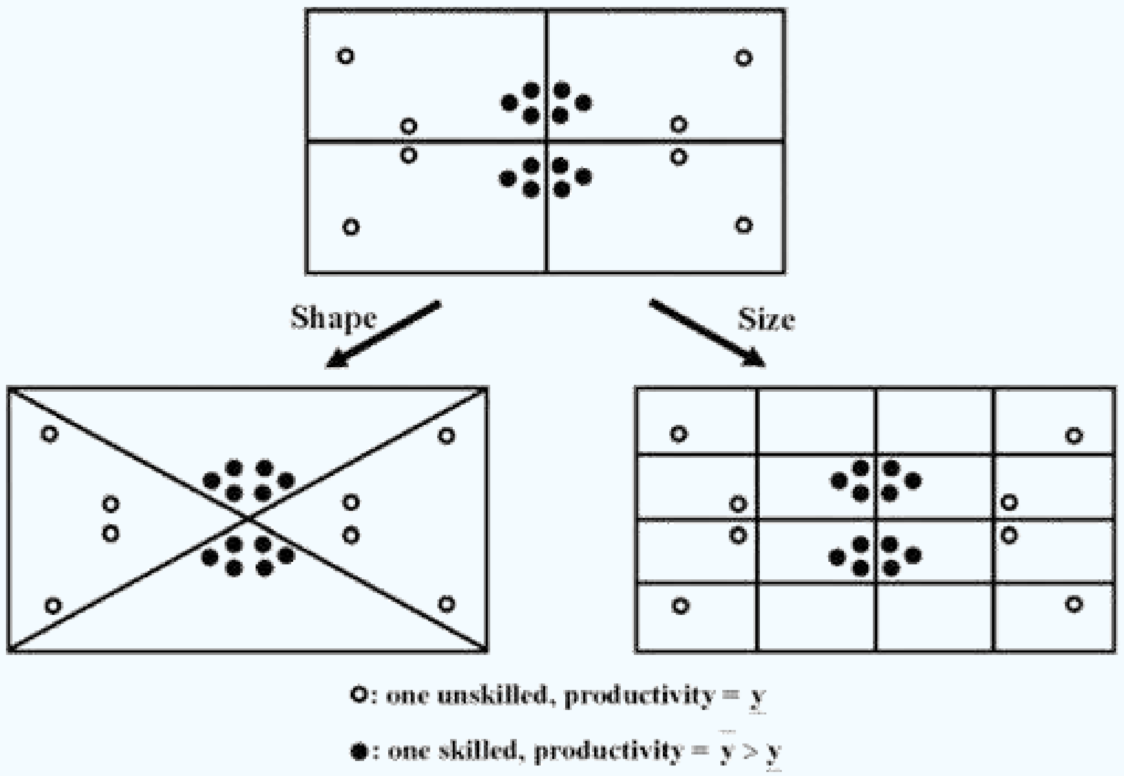
- Data is aggregated at
  - Postcode areas
  - Municipalities
  - Regions
  - Countries

- Problems:
  - Aggregation is often arbitrary
  - Areas are not of the same size

- This may lead to distortions
  - Modifiable areal unit problem (MAUP)

4. Modifiable areal unit problem

- **An illustration:**



Briant, Combes and Lafourcade (2010, JUE)

- **Aggregation seems to be important!**

4. Modifiable areal unit problem

- **Briant et al. (2010) investigate whether choice matters for regression results**

341 Employment Areas (EA)

341 Small squares (SS)

21 Régions (RE)

22 Large squares (LS)

4. Modifiable areal unit problem

- MAUP is of secondary importance
  - If y and x are aggregated in the same way
  - Matters more for larger areas (*e.g.* regions)
  - Use meaningful areas if possible

- Specification issues are much more important

5. Summary

Today:

- 'Space' in economics is becoming more and more important
- Incorporating space in econometric applications is not straightforward
- Important to define the spatial structure of the data
    - Spatial weight matrices
    - Modifiable areal unit problem

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5. Summary

Tomorrow:

- Spatial autocorrelation
- Spatial regressions

→ Subscribe to assignment groups before *Tuesday, 23:59*.

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Spatial econometrics (2)

Applied Econometrics for Spatial Economics

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tinbergen institute

- **This week**
  - **Learn about how to deal with spatial data**
  - **… and spatial econometrics**

- **Plan:**

Lecture #1:	Spatial data
Lecture #2:	Spatial autocorrelation and regressions
Lecture #3:	Spatial regressions (cont'd)

2. Spatial autocorrelation

- **Spatial autocorrelation between values**
  - **Implies** $\mathrm{cov}(x_i, x_j) = \mathrm{E}[x_i x_j] - \mathrm{E}[x_i] \cdot \mathrm{E}[x_j] \neq 0$
  - **Again,** j **refers to other locations**

- **Spatial autocorrelation, dependence, clustering**
  - **Fuzzy definitions in literature**

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2. Spatial autocorrelation

- **How to measure spatial autocorrelation**
  - **Moran's *I***
  - **Focus on one variable x (e.g. crime)**

- **H_0: independence, spatial randomness**
- **H_A: dependence**
  - **On the basis of adjacency, distance, hierarchy**

- **Moran's *I* is given by:**

$$I = \frac{R}{S_0} \times \frac{\tilde{\boldsymbol{x}}' \boldsymbol{W} \tilde{\boldsymbol{x}}}{\tilde{\boldsymbol{x}}' \tilde{\boldsymbol{x}}} \tag{4}$$

where R is the number of spatial units

S_0 is the sum of all elements of the spatial weight matrix

$\boldsymbol{W}$ is the spatial weight matrix

$\tilde{\boldsymbol{x}} = x - \bar{x}$ is a vector with the variable of interest

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2. Spatial autocorrelation

- **Moran's *I***
- **Use row-standardised spatial weight matrix W!**
- **Recall that** $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}}$ **(standardised** I**)**
  - **Note similarity with OLS:** $\hat{\beta} = \frac{x' y}{x' x}$
  - **Hence:** $W\tilde{x} = \alpha + I\tilde{x} + \epsilon$, **where** $\alpha = 0$
- **Moran's *I* is correlation coefficient (more or less)**
  - $\approx [-1,1]$
  - **But: expectation** $\mathrm{E}[I] = -\frac{1}{N-1}$
- **Visualisation**
  - **Moran scatterplot**

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2. Spatial autocorrelation

- **Moran's I**
- **How to investigate the statistical significance of (4)?**
  - $\frac{I - \mathrm{E}[I]}{\sqrt{\mathrm{var}[I]}}$ **(5)**
  - **However, $\sqrt{\mathrm{var}[I]}$ is difficult to derive**
  - $\mathrm{E}[I] = -1/(n-1)$
  - **Assume normal distribution of I to approximate $\sqrt{\mathrm{var}[I]}$ under H_0**
  - **Or: bootstrapping/simulation**

  - **See Cliff and Ord (1973) for more details**

2. Spatial autocorrelation

- **Moran's *I***

- **Also possible: correlation to other variables:**

$$I_S = \frac{\tilde{\boldsymbol{x}}' \boldsymbol{W} \tilde{\boldsymbol{z}}}{\tilde{\boldsymbol{x}}' \tilde{\boldsymbol{x}}}$$

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2. Spatial autocorrelation

- **How to calculate Moran's *I* using software**
  - SPAUTOC **in STATA**
  - SPLAGVAR **in STATA**
  - SPATIAL STATISTICS TOOLBOX **in ArcGIS**

- **Alternative: Getis and Ord's *G***
  - **Most of the time only Moran's *I* is reported**

2. Spatial autocorrelation

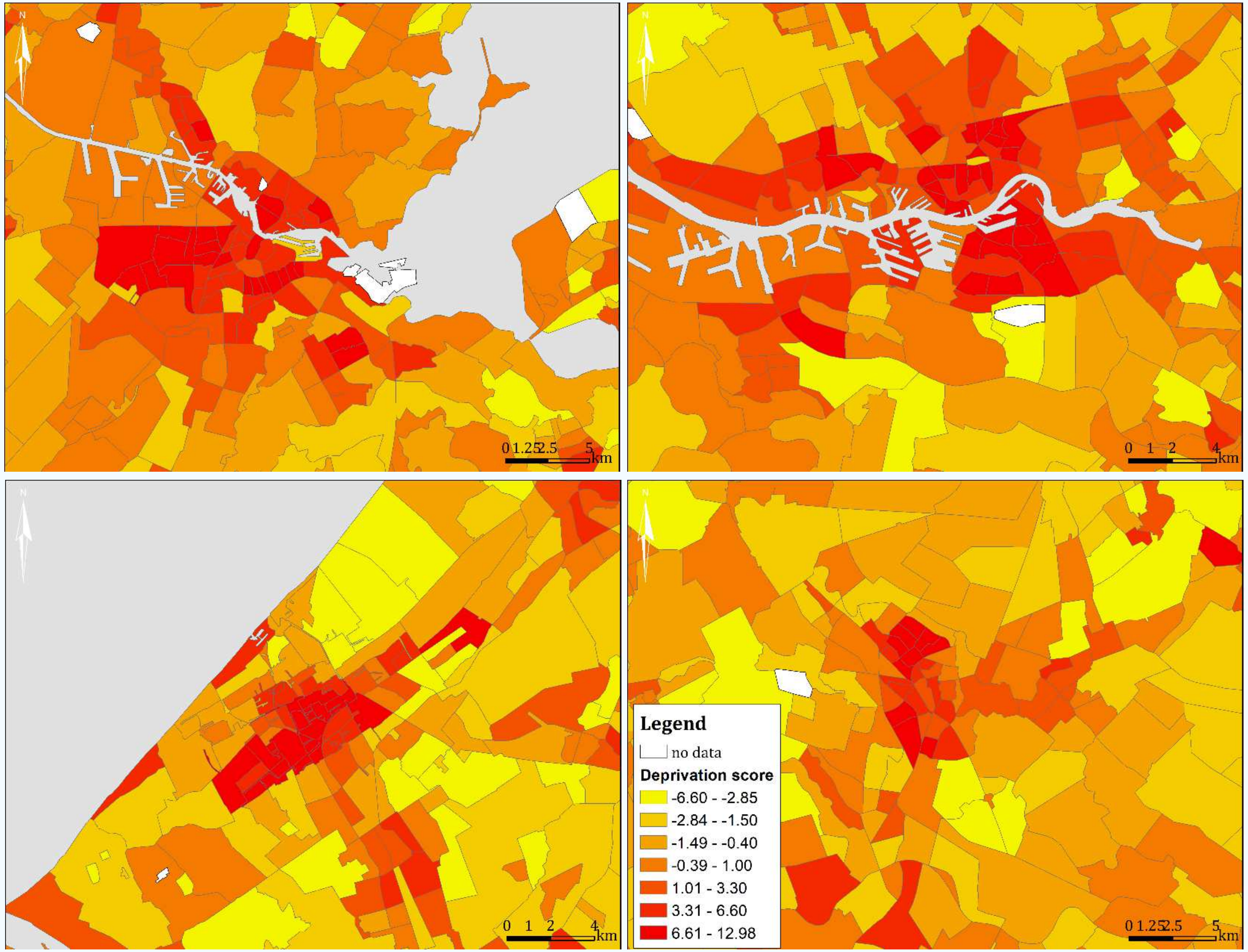
- **Let's try to answer the queston:**
  "Is social deprivation spatially clustered?"

- **How to determine the most deprived neighbourhoods?**

- **Dutch government calculated deprivation *z*-score for each neighbourhood**
  - **Based on housing quality, safety, perception and satisfaction**
  - ***Important:* the 83 most deprived neighbourhoods were selected for an investment of >€1 billion**

2. Spatial autocorrelation

Amsterdam **Rotterdam**



The Hague **Utrecht**

- **Determine spatial autocorrelation**
  1. **Determine distance between all neighbourhoods using centroids**
  2. **Use inverse distance function $w_{ij} = 1/\left(d_{ij}^{\gamma}\right)$ to determine spatial weights in weight matrix**
  3. **Calculate Moran's I: $\mathbf{W}\tilde{\mathbf{z}} = \alpha + I\tilde{\mathbf{z}} + \epsilon$ where $\tilde{\mathbf{z}} = \mathbf{z} - \bar{z}$ and W is a row-standardised weight matrix**
     - ***Note that $\mathbf{W}\tilde{\mathbf{z}}$ is a vector***
  4. **Bootstrap this procedure to estimate standard error (or use software)**

- **Determine spatial autocorrelation. *Note that***
  - **$\mathbf{W\tilde{z}}$ is a *vector***

$$\mathbf{W} \qquad\qquad \times \quad \tilde{\mathbf{z}} \quad = \mathbf{W\tilde{z}}$$

$$\begin{bmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{bmatrix} \times \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

  - **OLS: $\boldsymbol{\beta} = \frac{x'y}{x'x}$, while $I = \frac{\tilde{\mathbf{z}}'W\tilde{\mathbf{z}}}{\tilde{\mathbf{z}}'\tilde{\mathbf{z}}}$**
  - **Notation: $\frac{x'y}{x'x} = x^T y \left(x^T x\right)^{-1}$**

- Determine spatial autocorrelation
  1. Determine distance between all neighbourhoods using centroids
  2. Use inverse distance function $w_{ij} = 1/\left(d_{ij}^{\gamma}\right)$ to determine spatial weights in weight matrix
  3. Calculate Moran's I: $\mathrm{W}\tilde{\mathbf{z}} = \alpha + I\tilde{\mathbf{z}} + \epsilon$ where $\tilde{\boldsymbol{z}} = \boldsymbol{z} - \bar{z}$ and W is a row-standardised weight matrix
     - *Note that* $\mathrm{W}\tilde{\mathbf{z}}$ *is a vector*
  4. Bootstrap this procedure to estimate standard error (or use software)

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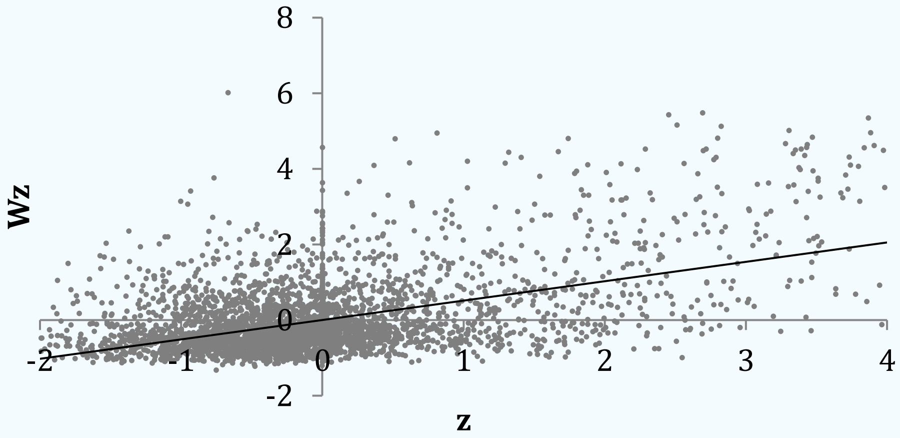
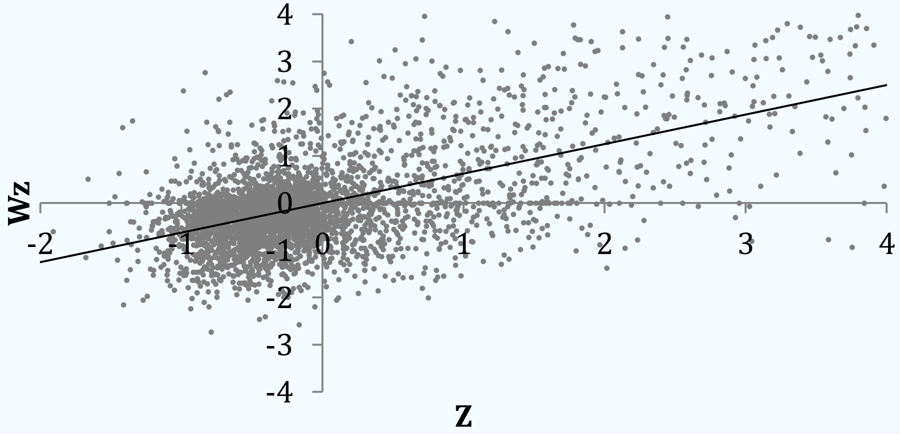
- **Calculate Moran's I**
  - **Using inverse distance function** $w_{ij} = \frac{1}{d_{ij}^{\gamma}}$

I = 0.513* *(γ=1)***



I = 0.625* *(γ=2)***



2. Spatial autocorrelation

- **Spatial correlation in deprivation**
  - **Local phenomenon?**
  - **You do not know *why* scores are autocorrelated…**
  - **No causal relationships!**

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2. Spatial autocorrelation

- It is important to make a distinction between *global* and *local* spatial autocorrelation
  - See Anselin (2003) for a discussion

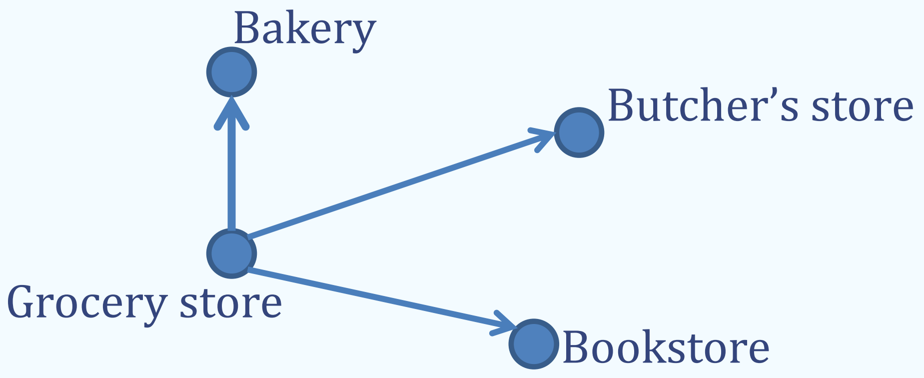
- Global spatial autocorrelation
  - Local shock affects the whole system

- Local spatial autocorrelation
  - Local shock only affects the 'neighbours'

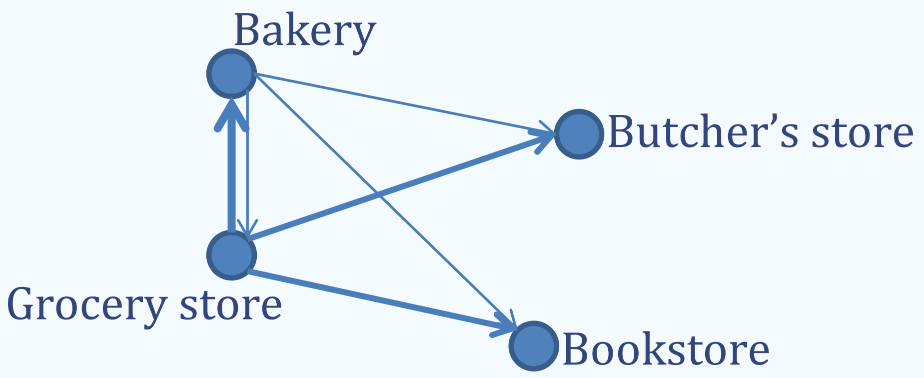
2. Spatial autocorrelation

- **Example: Consider an income increase for grocery store owner**

- **Local autocorrelation:**



- **Global autocorrelation:**



2. Spatial autocorrelation

- **Example: Consider an income increase for grocery store owner**
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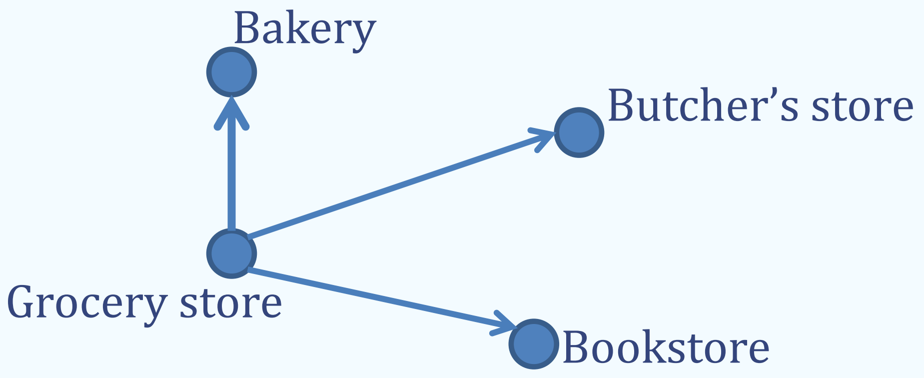
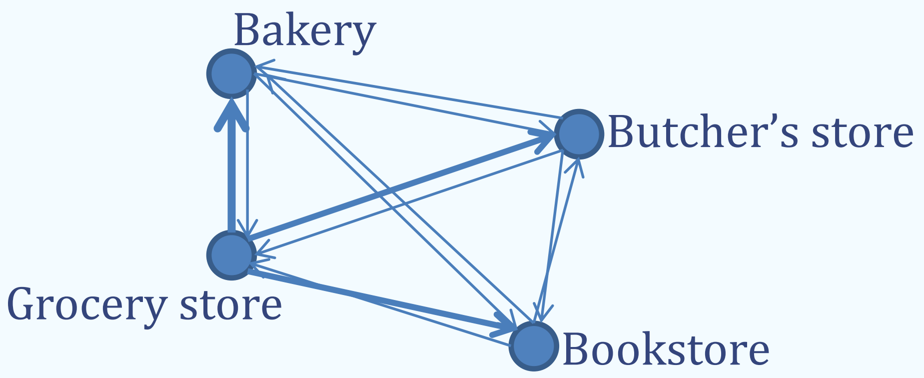


- **Global autocorrelation:**



… spatial multiplier process

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2. Spatial autocorrelation

- Let's define $z = \lambda W z + \mu$
  - Reduced-form of z is $z = [I - \lambda W]^{-1}\mu$
  - With $\lambda < 1$

- A Leontief expansion yields:
  - $[I - \lambda W]^{-1} = I + \lambda W + \lambda^2 W^2 + \lambda^3 W^3 + \cdots$

- $W^2 \rightarrow$ There is an impact of neighbours of neighbours (as defined in W) although it is smaller (λ^2)
  - Global autocorrelation
  - *Spatial multiplier* process
  - In practice: covariance may approach zero after a relatively small number of powers

2. Spatial autocorrelation

- Let's define $z = \lambda W\mu + \mu$
  - This is already a reduced-form of z
- No impact of behaviour beyond 'bands' of neighbours
  - Dependent on definition of W
  - …Local autocorrelation
- Covariance is zero beyond these bands

2. Spatial autocorrelation

- Local or global autocorrelation?
    - Dependent on application
    - Theory…

- **Taxonomy:**

$$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{W}\boldsymbol{X}\boldsymbol{\gamma} + \boldsymbol{\epsilon} \qquad (1)$$

with

$$\boldsymbol{\epsilon} = \lambda \boldsymbol{W}\boldsymbol{\epsilon} + \boldsymbol{\mu} \qquad (2)$$

W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

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- **Taxonomy:**

$$\boxed{\boldsymbol{y}} = \rho \boldsymbol{W}\boldsymbol{y} + \boxed{\boldsymbol{X}\boldsymbol{\beta}} + \boldsymbol{W}\boldsymbol{X}\boldsymbol{\gamma} + \boldsymbol{\epsilon} \qquad (1)$$

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- **Taxonomy:**

$$y = \rho W y + X\beta + \boxed{WX\gamma} + \epsilon \qquad (1)$$

with

$$\epsilon = \lambda W \epsilon + \mu \qquad (2)$$

W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

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- **Taxonomy:**

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with

$$\boldsymbol{\epsilon} = \boxed{\lambda \boldsymbol{W} \boldsymbol{\epsilon}} + \boldsymbol{\mu} \quad (2)$$

W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}$ (3)
  - $\rho \neq 0, \boldsymbol{\gamma} = 0, \lambda = 0$
  - **Spatial dependence in dependent variables**

- **Note similarity with time-series models**
  - **AR Model**
  - $\boldsymbol{y}_t = \rho \boldsymbol{y}_{t-1} + \boldsymbol{X}_t \boldsymbol{\beta} + \boldsymbol{\mu}_t$ (4)

- **Spatial lag model**
  - $$\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu} \qquad (3)$$

- **The outcome variable influences everyone (indirectly)**
  - **Global autocorrelation**

- **We may write**
  $(\mathbf{I} - \rho \boldsymbol{W})\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$
  $\boldsymbol{y} = (\boldsymbol{I} - \rho \boldsymbol{W})^{-1}(\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu})$ **with**
  $(\boldsymbol{I} - \rho \mathbf{W})^{-1} = \boldsymbol{I} + \rho \boldsymbol{W} + \rho^2 \boldsymbol{W}^2 + \rho^3 \boldsymbol{W}^3 + \cdots$

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- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu}$ (3)

- **We cannot estimate this by OLS because of reverse causality**

- **Recall AR-model:**

  $\boldsymbol{y}_t = \rho \boldsymbol{y}_{t-1} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu}_t$ (4)
  - **We can estimate this in principle by OLS because $\boldsymbol{y}_{t-1}$ is not influenced by $\boldsymbol{y}_t$**

- Spatial lag model

- Use maximum likelihood (ML) estimator
  - Selects the set of values of the model parameters that maximizes the likelihood function

- Instrumental variables (IV)
  - Instruments for y may be WX and $W^2\mathrm{X}^2$
  - Less efficient than ML, but feasible for 'large' datasets
  - *e.g.* Kelejian and Prucha (1998)

3. Spatial regressions

- **Spatial cross-regressive model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\gamma}\boldsymbol{W}\boldsymbol{X} + \boldsymbol{\mu}$ (5)
  - $\rho = 0, \boldsymbol{\gamma} \neq 0, \lambda = 0$

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- **Spatial cross-regressive model**
  - $y = X\beta + \gamma WX + \mu$ (5)

- **Include (transformations) of exogenous variables in the regression**
  - **OLS is fine!**

- **Autocorrelation is local**

- **Spatial error model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},\ \text{with } \boldsymbol{\epsilon} = \lambda\mathbf{W}\boldsymbol{\epsilon} + \boldsymbol{\mu}$ (6)
  - $\rho = 0, \boldsymbol{\gamma} = 0, \lambda \neq 0$

- **Spatial error model**
  - $y = X\beta + \epsilon, \text{ with } \epsilon = \lambda W\epsilon + \mu$ (6)

- **Omitted spatially correlated variables**
  - e.g. Ad-hoc defined boundaries
  - Uncorrelated to X!

- **Consistent estimation of parameters β**
- **But: inefficient**
  - ϵ are not i.i.d.
  - Standard errors are higher in OLS
  - β may be different in 'small' samples

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- **How to apply these models in practice?**

- SPAUTOREG **in STATA**
- SPATREG **in STATA**
- **GeoDa (free software, also for large datasets)**
- PACE'S SPATIAL STATISTICS TOOLBOX **in MATLAB**

Today:

- Test spatial autocorrelation using Moran's *I*
- Local vs. global spatial autocorrelation
- Incorporate space in regression framework
- Spatial regressions
  - Spatial lag model
  - Spatial cross-regressive model
  - Spatial error model

Tomorrow:

- Spatial regressions (extensions)
- When (not) to use spatial econometrics?

→ **Subscribe to assignment groups before *Tonight, 23:59*.**

Spatial econometrics (2)

Applied Econometrics for Spatial Economics

Hans Koster

Professor of Urban Economics and Real Estate

Spatial econometrics (3)

Applied Econometrics for Spatial Economics

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1. Introduction

- This week
  - Learn about how to deal with spatial data
  - … and spatial econometrics

- Plan:

Lecture #1:	Spatial data
Lecture #2:	Spatial autocorrelation and regressions
Lecture #3:	Spatial regressions (cont'd)
	When (not) to use spatial econometrics?

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2. Spatial regressions (continued)

- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu}$ (3)
  - $\rho \neq 0, \boldsymbol{\gamma} = 0, \lambda = 0$
  - **Spatial dependence in dependent variables**

- **Spatial cross-regressive model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \gamma \boldsymbol{W}\boldsymbol{X} + \boldsymbol{\mu}$ (5)

- **Spatial error model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, **with** $\boldsymbol{\epsilon} = \lambda \mathbf{W}\boldsymbol{\epsilon} + \boldsymbol{\mu}$ (6)

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2. Spatial regressions (continued)

- **Three issues are on the table**
  1. **When should you use these models?**
  2. **Which of the models should you choose?**
  3. **Can we combine these different spatial models?**

1. When should you use these models?

- **Test for spatial effects**
  - **H_0: No spatial dependence**

- **Estimate standard OLS, $y = X\beta + \epsilon$**
  - **Calculate Moran's I using $\hat{\epsilon}$**
  - $I = \frac{R}{S_0} \times \frac{\hat{\epsilon}' W \hat{\epsilon}}{\hat{\epsilon}' \hat{\epsilon}}$

- **Moran's I does have a rather uninformative alternative hypothesis**
  - **H_A: Spatial dependence…**

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1. When should you use these models?

- **Lagrange multiplier tests provide more information**
  - LM_ρ - test for presence of spatial lag
  - LM_λ - test for presence of spatial error

2. Spatial regressions (continued)

1. When should you use these models?

- **Test for spatial lag**
  1. **Run OLS**
  2. **Run LM_ρ-test**

$\mathbf{H_0}$: $\rho = 0$
$\mathbf{H_A}$: $\rho \neq 0$

$$LM_\lambda = \frac{1}{\mathrm{n}J}\left(\frac{\epsilon' W y}{s^2}\right)^2 \sim \chi_1^2 \qquad (9)$$

with $\boldsymbol{J} = [(\boldsymbol{WX\beta})'\boldsymbol{M}(\boldsymbol{WX\beta}) + \boldsymbol{T}s^2]/n\boldsymbol{s}^2$ **and**
$\mathbf{M} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$

1. When should you use these models?

- **Test for spatial error**
  1. **Run OLS**
  2. **Run LM_λ-test**

$\mathbf{H_0}$: $\lambda = 0$
$\mathbf{H_A}$: $\lambda \neq 0$

$$LM_\lambda = \frac{1}{\mathrm{T}}\left(\frac{\epsilon' W \epsilon}{s^2}\right)^2 \sim \chi_1^2 \quad (8)$$

with $\mathrm{T} = \mathrm{tr}\left((\mathbf{W}' + \mathbf{W})W\right)$ **and** $s = \epsilon'\epsilon\,/n$

1. When should you use these models?

- **However,**
  - **Spatial errors and lags may be correlated**
  - **May also be both present**

- **Use *robust* LM tests**
  - LM_ρ^* **adds correction factor for potential spatial error**
  - LM_λ^* **adds correction factor for potential spatial lag**
  - **Complex formulae!**

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2. Which of the models should you choose?

- Estimate *robust* LM tests using software

- Common practice
  - Choose and estimate the model for which the statistic is the most significant

2. Which of the models should you choose?

- Estimate *robust* LM tests using software
- Common practice
  - Choose and estimate the model for which the statistic is the most significant
- When LM_{λ}^{*} and LM_{ρ}^{*} are statistically insignificant we may use OLS
- Robust LM-tests are typically provided in STATA output

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3. Can we combine these spatial models?

- In practice, both a spatial lag and spatial error may be present

- How to estimate?
  - Use Kelejian and Prucha's GS2SLS method
  - Three-stage procedure!

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3. Can we combine these spatial models?

- **Complicated stuff!**

- **Let software do the difficult calculations!**
  - SPAUTOREG **in STATA**
  - SPIVREG **in STATA**

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3. Mostly pointless spatial econometrics?

- Gibbons and Overman (2012)
  - *"Mostly pointless spatial econometrics?"*
  - Nice read!

- We are interested to identify causal impacts $\boldsymbol{\beta}$:

  $y = X\beta + \mu$

- Typical features of spatial data
  - Unobserved variables correlated with X
  - Omitted variable bias!
  - Large datasets

3. Mostly pointless spatial econometrics?

- **Tempting to 'fix' omitted variable bias by including a spatial lag**

- **Let's consider again:**
  $\boldsymbol{y} = \rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{\mu}$

- **Reduced-form:**
  $\boldsymbol{y} = \rho \boldsymbol{W}(\rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{\mu}) + \boldsymbol{X\beta} + \boldsymbol{\mu}$
  $\boldsymbol{y} = \rho \boldsymbol{W}(\rho \boldsymbol{W}(\rho \boldsymbol{W y} + \boldsymbol{X\beta} + \boldsymbol{\mu}) + \boldsymbol{X\beta} + \boldsymbol{\mu}) + \boldsymbol{X\beta} + \boldsymbol{\mu}$
  …
  $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{WX}\pi_1 + \boldsymbol{W}^2\boldsymbol{X}\pi_2 + \boldsymbol{W}^3\boldsymbol{X}\pi_3 + [\dots] + \tilde{\boldsymbol{\mu}}$

… The last equation suggests that in the end y is just a non-linear function of the X-variables

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3. Mostly pointless spatial econometrics?

- Reduced form of spatial lag ≈ spatial cross-regressive model
  - It is hard to prove that spatial lag is 'right' model
  - So, it is hard to distinguish empirically between the two types of models
  - Only when there is a structural (network) model, a spatial lag may be appropriate

3. Mostly pointless spatial econometrics?

- **The spatial lag model *does not* solve the problem of omitted variable bias!**
  - Think of real exogenous sources of variation in X to identify β
  - Use instruments or quasi-experiments
  - More discussion on identification strategies in last week!

- Estimate spatial error model?
  - Spatial datasets are typically large
  - Efficiency issues are *usually* not so important

3. Mostly pointless spatial econometrics?

- **Why then use spatial econometrics?**
  1. **Exploratory tool to investigate spatial autocorrelation**
  2. **Test for spatial dependence and heterogeneity, also in quasi-experiments and when using instruments**
  3. **Investigate whether results are robust to spatial autocorrelation (using different W)**
  4. **Spatial cross-regressive models are often useful and straightforward to interpret**

Today:

- Spatial regressions
  - Spatial lag model
  - Spatial cross-regressive model
  - Spatial error model
- Use robust LM tests to distinguish between different types
  - Or: combine using advanced methods
- Spatial econometrics are a useful tool, *but* not a way to identify causal effects

Next week:

- Discrete choice methods and the random utility framework
- Revealed vs. stated preference data

Spatial econometrics (3)

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