

# Discrete choice (1)

Applied Econometrics for Spatial Economics

**Hans Koster**

*Professor of Urban Economics and Real Estate*

1. Introduction
2. The RUM framework
3. Value of time
4. Multiple alternatives
5. Summary

- **Today:**
  1. Spatial econometrics
  2. **Discrete choice**
  3. **Identification**
- **Tomorrow:**
  4. **Hedonic pricing**
  5. **Quantitative spatial economics**

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- **Today:**

1. Spatial econometrics

2. **Discrete choice**

- **Random utility framework, estimating binary and multinomial regression models**

3. **Identification**

- **Tomorrow:**

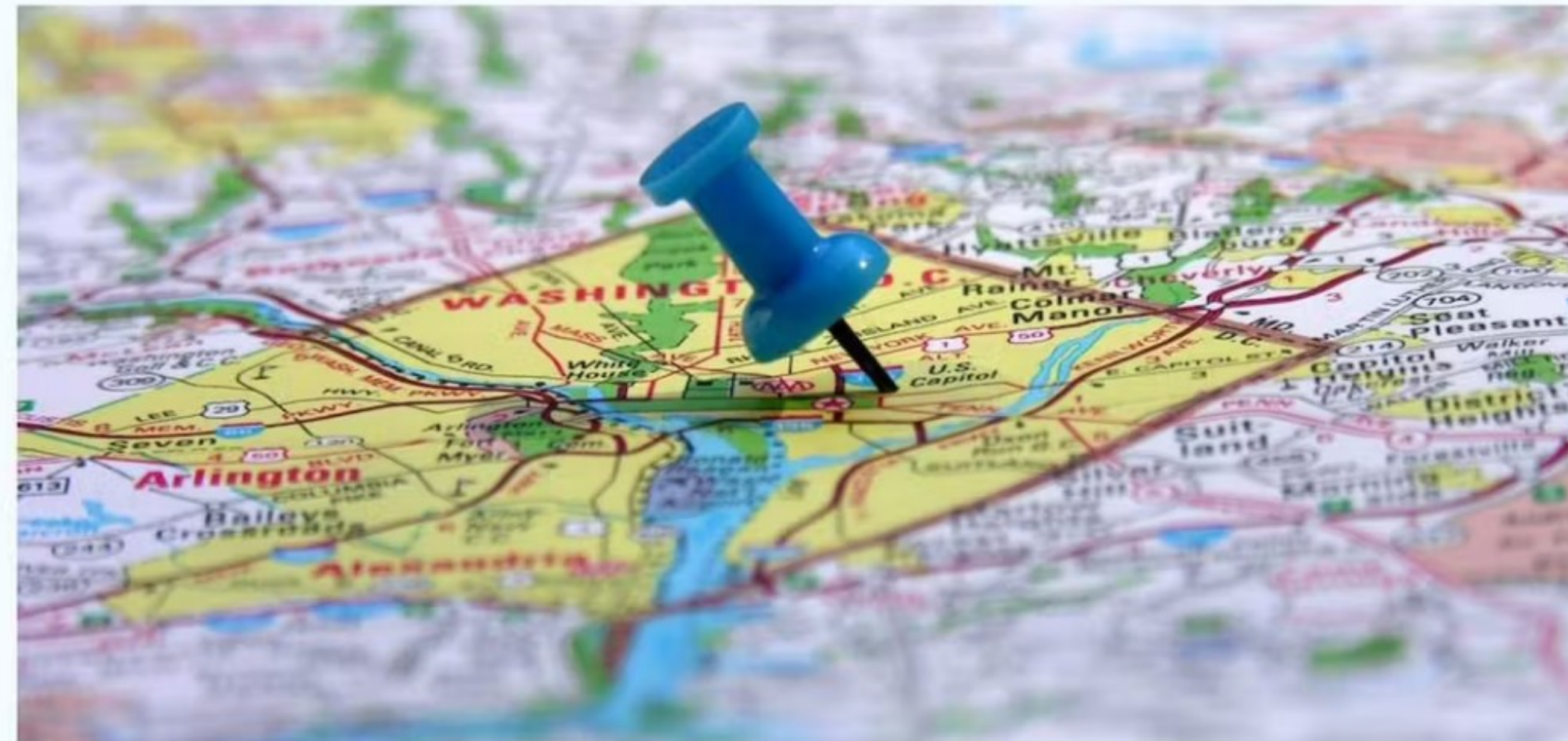
4. **Hedonic pricing**

5. **Quantitative spatial economics**

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- ***Continuous* choice: estimate marginal changes in behaviour**
  - E.g. “when fuel price increases by 10%, the demand for fuel will decrease by 2%”
  - Standard micro-economic theory applies
  
- **Transport demand often has a discrete (binary) nature**
  - Some  $x$  impacts a discrete  $y$
  - Then use discrete choice methods

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- **Discrete choice methods**
  - *Dependent variable  $y_i$  is discrete*
- **Why not use OLS?**
- **Let's have the standard OLS equation**  
$$y_i = \beta x_i + \epsilon_i \tag{1}$$
**where  $i$  indexes the individual**

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- **OLS may be consistent for binary choice**
  - *But,  $y_i$  (and therefore  $\epsilon$ ) is not normally distributed*
  
- **Horrace and Oaxaca (2006)**
  - Leads to biased and inconsistent estimates if  $\hat{y}_i$  lies 'often' outside the  $[0,1]$  interval
  - I show later today why that is an issue...
  
- **OLS does not necessarily provides a link with economic theory**
  
- **Not suitable for multinomial choice**



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- **Indirect utility may be given by:**

$$U_{iA} = V_A(\text{travel time}_A) + \epsilon_{iA} \quad (2)$$

$$U_{iB} = V_B(\text{travel time}_B) + \epsilon_{iB} \quad (3)$$

- $V_A, V_B \rightarrow$  **deterministic utility**

- **Random terms:  $\epsilon_{iA}, \epsilon_{iB}$ : random taste variation**

- **Random utility model (RUM)**
- **Note that the levels of  $U_{iA}$  and  $U_{iB}$  are not directly observed!**

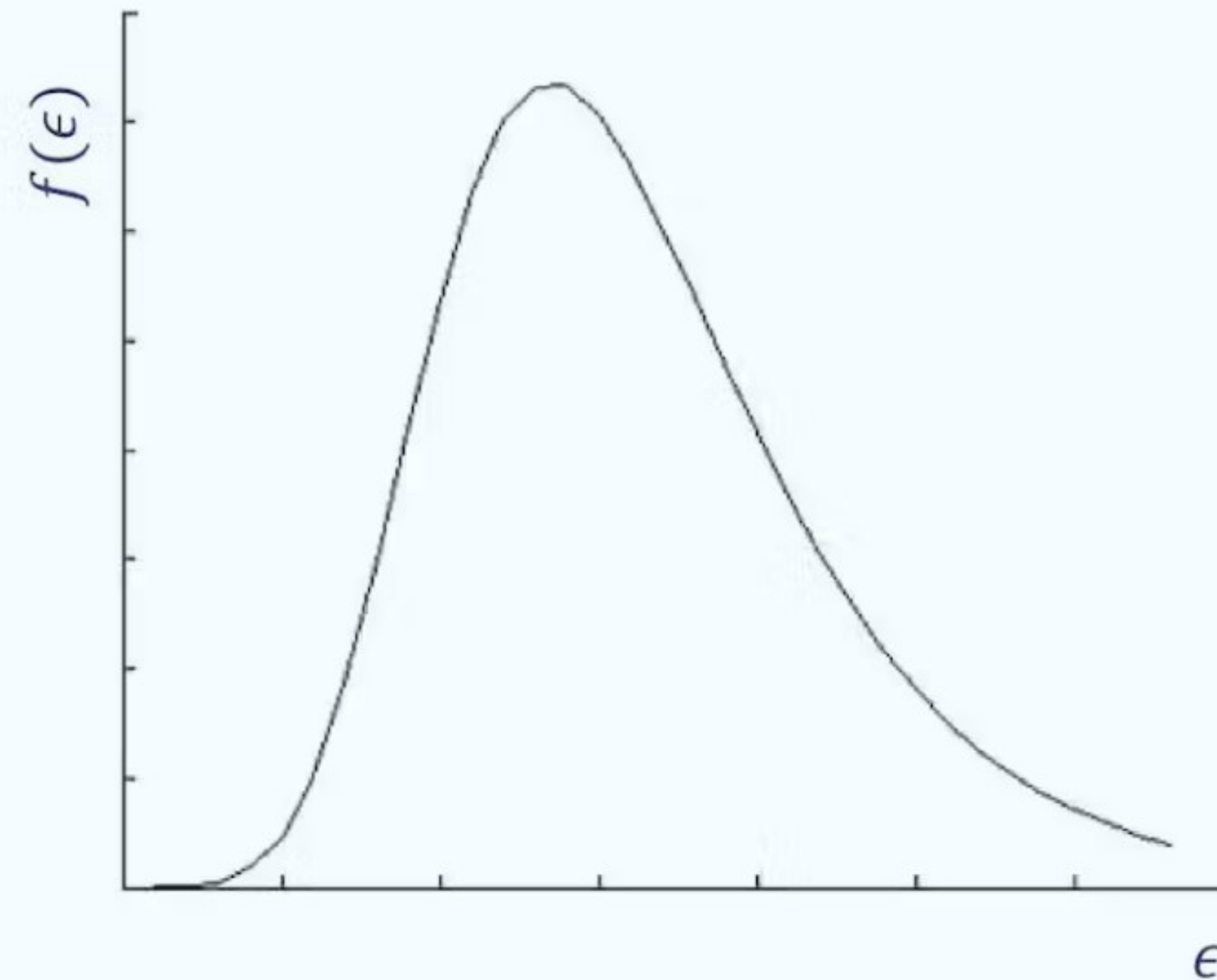
- $\Pr(Y = A) = \Pr(U_{iA} > U_{iB})$
- $\Pr(V_A + \epsilon_{iA} > V_B + \epsilon_{iB}) = \Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$

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- **Two things are unknown**
  - **Which distribution for  $\epsilon$ 's?**
  - **What is the functional form for  $V_A$  and  $V_B$ ?**

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- **Which distribution for  $\epsilon$ 's?**
  - $\epsilon$ 's are unobserved
  - You draw them from a distribution
  - Logit: Extreme Value Type I distribution



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- **Which distribution for  $\epsilon$ 's?**
  - **Extreme Value Type I distribution**
  - **Generates simple closed-form solutions!**  
→  $\Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$
  - **Daniel McFadden (1964)**



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- **It appears that:**

$$\Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} \quad (4)$$

- **With two alternatives this can be written as:**

$$\Pr(Y = A) = \frac{1}{1 + e^{V_B - V_A}}$$

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- **Which functional form for  $V_A$  and  $V_B$ ?**
  - **Can be any function**
  - **Linear function is often assumed**
  - **Can be extended with multiple variables**

$$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \epsilon_{jA} \quad (5)$$

$$U_{jB} = \beta p_{jB} + \kappa t_{jB} + \epsilon_{jB} \quad (6)$$

where  $p_{jA}$  is the price of a trip and  $t_{jA}$  is travel time of alternative  $j$

- $\beta < 0, \kappa < 0$

- **Recall (from previous slide):**

- $$\Pr(Y = A) = \frac{1}{1 + e^{\beta(p_{jB} - p_{jA}) + \kappa(t_{jB} - t_{jA})}}$$

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- **Important concept in Transport Economics:**  
*Value of Time (VOT)*
  - “How much are you willing to pay to reduce your travel time with one hour, *holding utility constant*”
  
- **Let's take the deterministic utility function**  
$$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \varepsilon_{jA} \quad (7)$$
  
- **When  $t_{jA}$  is measured in hours, the VOT can be written as  $\kappa/\beta$**

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- **Important concept in Transport Economics:**  
**Value of Time (VOT)**
  - “How much are you willing to pay to reduce your travel time with one hour, *holding utility constant*”
  
- **Let's take the deterministic utility function**  
$$U_{iA} = \beta p_{iA} + \kappa t_{iA} + \gamma x_{iA} + \varepsilon_{iA}$$
  - $\Delta U_{iA} = 0$  (7)
  
- **Show that when  $t_{iA}$  is measured in hours, the VOT can be written as  $\kappa/\beta$**



With  $U_{iA} = \beta p_{iA} + \kappa t_{iA} + \gamma x_{iA} + \varepsilon_{iA}$ , show that when  $t_{iA}$  is measured in hours, the Value of Time can be written as  $\kappa/\beta$ .



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→ Show that when  $t_{jA}$  is measured in hours, the VOT can be written as  $\kappa/\beta$

Let's look at a change in utility:

$$\Delta U_{iA} = \beta \Delta p_{iA} + \kappa \Delta t_{iA} + \Delta \varepsilon_{iA}$$

- We hold utility constant, so  $\Delta U_{iA} = 0$
- Because  $E[\varepsilon_{iA}] = 0$ ,  $\Delta \varepsilon_{iA} = 0$
- $-\beta \Delta p_{iA} = \kappa \Delta t_{iA}$
- $\Delta t_{iA} = -1$  (WTP for one hour reduction in travel time)
- $\Delta p_{iA} = \frac{\kappa}{\beta}$

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- Value of time is often used in cost benefit analyses
- VOT depends on trip purpose
  - Business €26.25/h
  - Commuting €9.25/h
  - Social purpose €7.50/h
- VOT depends on income
  - About 50% of net income

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- **Other applications**
  - **Value of a Statistical Life (VSL)**  
*The VSL is the local tradeoff between fatality risk and money*
  - **Value of schedule delay (VSD)**  
*The VSD is the local tradeoff between being too early/late and money*
  - **Etc.**
  - **... What is necessary is a cost/reward parameter in the discrete choice experiment**

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- **The choice probability for two alternatives:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B}}$$

- Usually there are more alternatives in the choice set

- Train, bus, car
- Rotterdam, Antwerp, Hamburg
- Routes to the VU

- **Simply extend the logit formula:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}}$$

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- The aggregate utility derived from the choice set is summarised by the logsum:

$$E[CS] = \frac{1}{v} \ln(e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C})$$

- $v$  is the marginal utility of income
  - Can be used in welfare estimates
- 
- Assume  $\beta x_A = \beta x_B = 10$
  - Now alternative  $C$  is added and  $\beta x_C = 1$
  - The average utility per alternative decreases from 10 to 7 but  $E[CS]$  increase
    - ‘Love of variety’ effect

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- **Property of logit formula:**
  - The *ratios* of choice probabilities for A and B do not depend on whether or not C is in the choice set
  - Independence of irrelevant alternatives

- $$\frac{\Pr(Y=A)}{\Pr(Y=B)} = \frac{\left( \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}} \right)}{\left( \frac{e^{\beta x_B}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}} \right)} = \frac{e^{\beta x_A}}{e^{\beta x_B}}$$

- **Let's find out whether this is a desirable property...**

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- **'The *Red Bus-Blue Bus*' problem**
- **Choice set 1: Train, red bus, blue bus**
- **Assume market shares are 70, 15 and 15%**

	Train	Red bus	Blue bus
V	2.54	1	1
Prob	0.700	0.150	0.150

- **Choice set 2: Train, red bus, so:**

	Train	Red bus
V	2.54	1
Prob	0.823	0.177

- **Probability to take the bus in choice set 2 is**

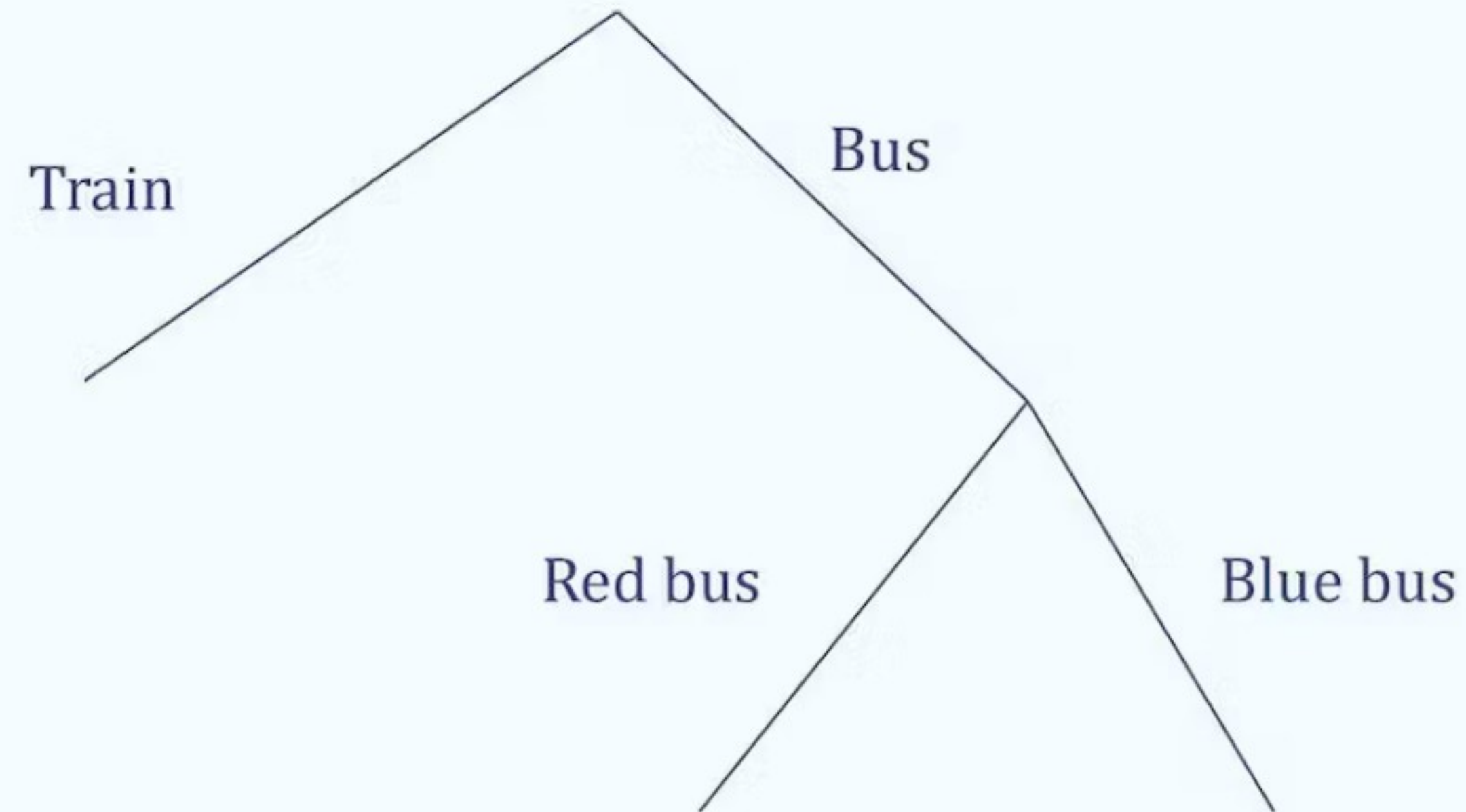
$$\frac{e^1}{e^{2.54} + e^1} = 0.177$$

- **Higher probability – not very realistic as red buses and blue buses are identical**



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- **So, when some alternatives are more similar than other alternatives, the use of multinomial choice model may be misleading**
- **Use nested logit!**



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- **Nested logit takes into account correlation between alternatives**
  - **But define nests yourself!**

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- **Let us define utility as follows:**

$$U_{jg} = V_j + W_g + \epsilon_{jg}$$

$V_j$  only differs within nests between alternatives  $j$

$W_g$  only differs between nests  $g$

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- **We may write the probability to choose an alternative:**
  - $\Pr(d_j = 1) = \Pr(g) \cdot \Pr(j | g)$
  - $\Pr(j | g) = \frac{e^{V_j/\lambda_g}}{\sum_{k \in g} e^{V_k/\lambda_g}}$
  - $\Pr(g) = \frac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}}$   
**with**  $I_g = \log(\sum_{j \in g} e^{V_j/\lambda_g})$
- $\lambda_g = 1 \Rightarrow$  **no correlation (multinomial logit)**
- $\lambda_g \rightarrow 0 \Rightarrow$  **perfect correlation (red bus/blue bus)**

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- **When  $j$  and  $k$  are in the same nest:**

$$\frac{\Pr(d_j = 1)}{\Pr(d_k = 1)} = \frac{e^{W_g + V_j} / \lambda_g}{e^{W_g + V_k} / \lambda_g} = \frac{e^{W_g + V_j}}{e^{W_g + V_k}} = \frac{e^{V_j}}{e^{V_k}}$$

- **IIA property holds *within* nests**

- **When  $\lambda_g \rightarrow 0$ :**

- $\Pr(j | g) = \frac{e^{V_j / \lambda_g}}{\sum_{k \in g} e^{V_k / \lambda_g}} = 1 / k_g$

- $\Pr(g) = \frac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}} = \frac{e^{W_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}}}}$

- **Hence, multinomial logit *between* nests**

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- **So, nested logit probability depends on**
  - **Probability to choose a nest**
  - **Probability to choose an alternative within the nest**
  
- **Note that Nested Logit does not imply a *sequential* choice**

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# Discrete choice (2)

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- **How to estimate binary discrete choice models?**
- **Three main options**
  1. **Linear probability model**
  2. **Logit**
  3. **Probit**

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### Advantages:

- **Consistent when  $0 \leq \hat{y}_j \leq 1 \forall j$**
- **Easy to interpret**
  - $\frac{\partial \Pr(d_j=1)}{\partial x} = \beta$
- **Computationally feasible**
  - **Important for large panel datasets**
- **In practice, leads to very similar results as Logit and Probit**

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### Disadvantages:

- **No direct link with structural parameters of utility function**
  - e.g. not able to calculate aggregate utility from choice set
  
- **Biased for small samples and possibly inconsistent marginal effects**
  - **Linearity?**
  
- **Not suitable for multinomial choices**

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- **Let's define**

$$\Pr(d_j = 1) = \frac{1}{1 + e^{-\beta' x_j}}$$

- **Example: regress 0/1 variable on *differences* in characteristics of the alternatives**

Chosen <sub>B</sub>	Price <sub>B</sub> -Price <sub>A</sub>	Time <sub>B</sub> -Time <sub>A</sub>
1	-14	5
0	5	0
0	15	-20
1	-8	13
1	-10	3
1	3	-5
0	20	10

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- **Recall**

- $\Pr(d_j = 1) = \left(1 + e^{-\beta' x_j}\right)^{-1}$

→ **Calculate the marginal effect on probability of one unit increase in  $x$  (so  $\frac{\partial \Pr(d_j=1)}{\partial x_j}$ )**

What is the marginal effect on the probability of one

unit increase in  $x$   $\left( \frac{\partial \Pr(d_j = 1)}{\partial x_j} \right)$



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- **Marginal effects:**

- **Use chain rule of differentiation**

- $$\frac{\partial \Pr(d_j=1)}{\partial x_j} = -\left(1 + e^{-\beta'x_j}\right)^{-2} \times e^{-\beta'x_j} \times -\beta$$

- $$\frac{\partial \Pr(d_j=1)}{\partial x_j} = \beta \frac{e^{-\beta'x_j}}{(1+e^{-\beta'x_j})^2}$$

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- **The change in the probability for one unit increase in  $x$**

- $$\frac{\partial \Pr(d_j=1)}{\partial x_j} = \beta \frac{e^{-\beta' x_j}}{(1+e^{-\beta' x_j})^2}$$

- **Marginal effect depends on  $x_j$ , so is not constant/linear**
  - **For example, evaluate at mean values of  $x$**



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- **Software**
  - LOGIT **or** LOGISTIC **in STATA**
  - REGRESSION – BINARY LOGISTIC **in SPSS**
  
- **In STATA you can select to report marginal effects**
  - **Use** MARGINS **after** LOGIT **command**
  - **Choose at which  $x$  the values are evaluated (e.g. at means)**

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### Advantages of Logit:

- **Predicted probability is always between one and zero**
- **Clear link to random utility framework**
  - **Log-sum may be used for welfare calculations**
- **Closed-form marginal effects**
  - **Usually leads to very similar results as Probit**
- **Can include 'fixed effects' (XTLOGIT in STATA)**
  - *e.g. to control for individual heterogeneity*

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### Disadvantages of Logit:

- Why Extreme Value Type I distribution for  $\epsilon$ ?
  
  
  
  
  
  
  
  
  
  
- Maximum likelihood / non linear model

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- **We may also assume that  $\epsilon_j$  is normally distributed, so  $\epsilon_j = N(0, \sigma^2)$** 
  - **This implies  $\Pr(d_j = 1) = \Phi(\beta' x_j)$**
  - **However, no closed-form for cumulative normal distribution!**

- **Marginal effects:**

$$\frac{\partial \Pr(d_j=1)}{\partial x_j} = \beta \phi(\beta x_j)$$

**where  $\phi(\cdot)$  is the density function of the normal distribution**

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### Advantages:

- Normal distribution for  $\epsilon_j$  may seem more reasonable
- Probability is always between one and zero

### Disadvantages:

- No closed-form marginal effects
- Hard to include many fixed effects

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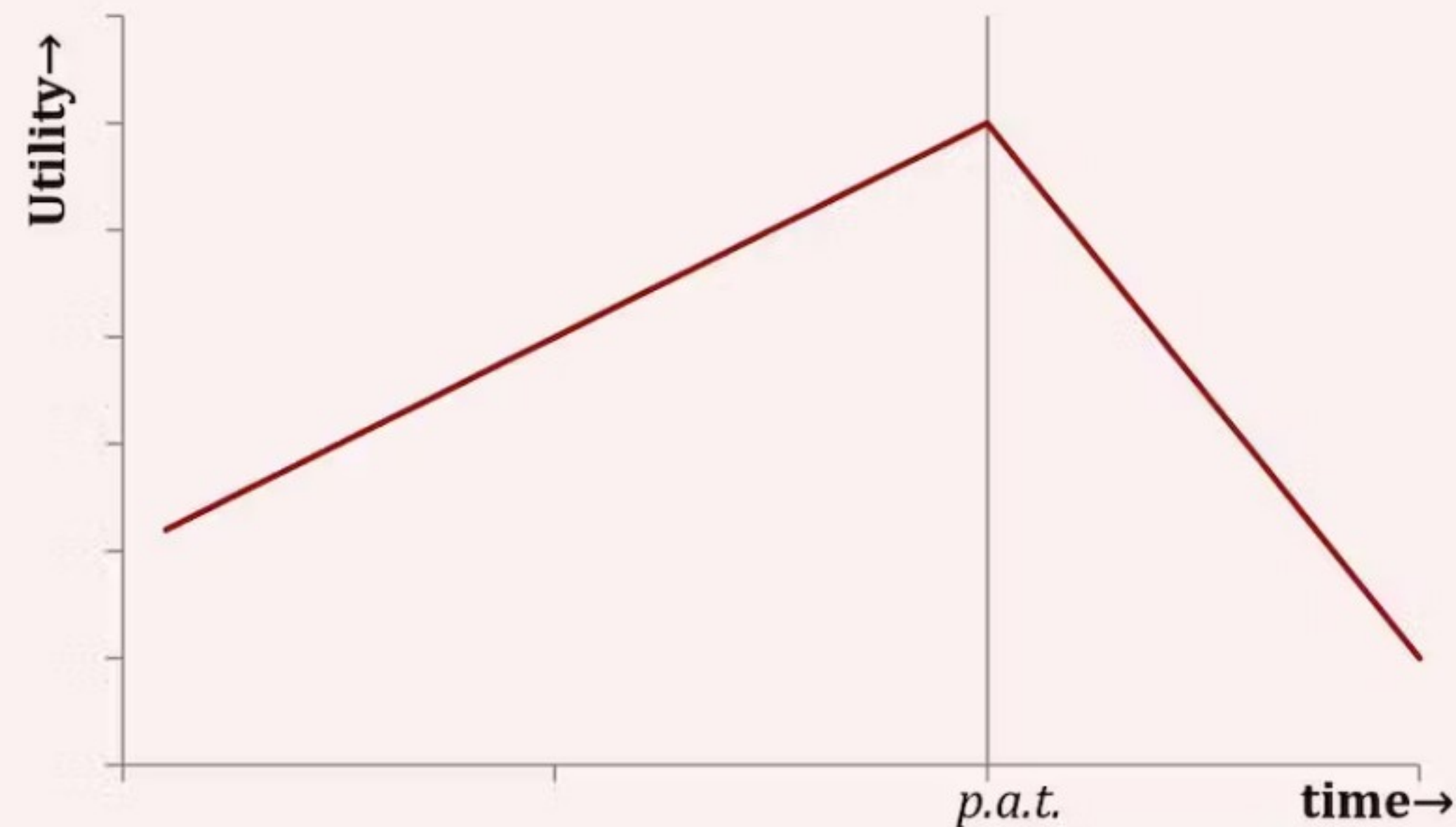
- **How to choose between the three models?**
  - **Probit estimates  $\approx$  Logit estimates**
  - **Check for robustness of marginal effects**
  - **Large sample and interested in marginal effects?**
    - **Usually linear probability model!**
    - **There is an ongoing debate in economics on this issue**

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- **Koster and Koster (2014)**
  - **Estimate the value of time and unreliability**
  - **Uses a stated choice experiment**
  
- **Stated-choice experiment about preferences of morning commuters**
  - **“*Spitsmijden*” (Peak-avoidance project)**
  - **People get a reward if they avoid the peak**
  
  - **But: they may be too early or late at work!**
  - **Trade-off**

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- Value of time
- Value of schedule delay early
- Value of schedule delay late
  - $p.a.t.$  = preferred arrival time





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- **Example of a choice with two alternatives and uncertainty**

Your preferred arrival time if there is no delay is: 8:40.

	Alternative 1		Alternative 2	
Departure time from home	6:05		6:50	
Probability	80%	20%	90%	10%
Total travel time	30 min	40 min	20 min	35 min
Arrival time at work	6:35	6:45	7:10	7:25
Reward	4 euro	4 euro	0 euro	0 euro

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- **Utility is specified as follows**

- $$U_{icj} = \beta^R R_{icj} + \beta^T T_{icj} + \beta^{SDE} SDE_{icj} + \beta^{SDL} SDL_{icj} + \epsilon_{icj}$$

$i$       **individual**

$c$       **choice**

$j$       **alternative**

$R_{icj}$     **expected *reward***

$T_{icj}$     **expected travel time**

$SDE_{icj}$  **expected time before *p.a.t.***

$SDL_{icj}$  **expected time after *p.a.t.***

$\epsilon_{icj}$     **random taste variation, Extreme Value Type I distributed**

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■ **Utility is specified as follows**

$$\bullet \quad U_{icj} = \beta^R R_{icj} + \beta^T T_{icj} + \beta^{SDE} SDE_{icj} + \beta^{SDL} SDL_{icj} + \epsilon_{icj}$$

$$\bullet \quad \Delta u_{inj} = 0 = \beta^R \Delta R_{icj} + \beta^T \Delta T_{icj} + \beta^{SDE} \Delta SDE_{icj} + \beta^{SDL} \Delta SDL_{icj}$$

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- **Estimate the value of time and unreliability**

- **Value of time (VOT):**

$$-\beta^R \Delta R_{icj} = \beta^T \Delta T_{icj} \rightarrow \Delta T_{icj} = -1 \rightarrow -\Delta R_{icj} = -\frac{\beta^T}{\beta^R}$$

*Note that we look at the willingness to pay. Because the experiment focuses on rewards, we have  $-\Delta R_{icj}$*

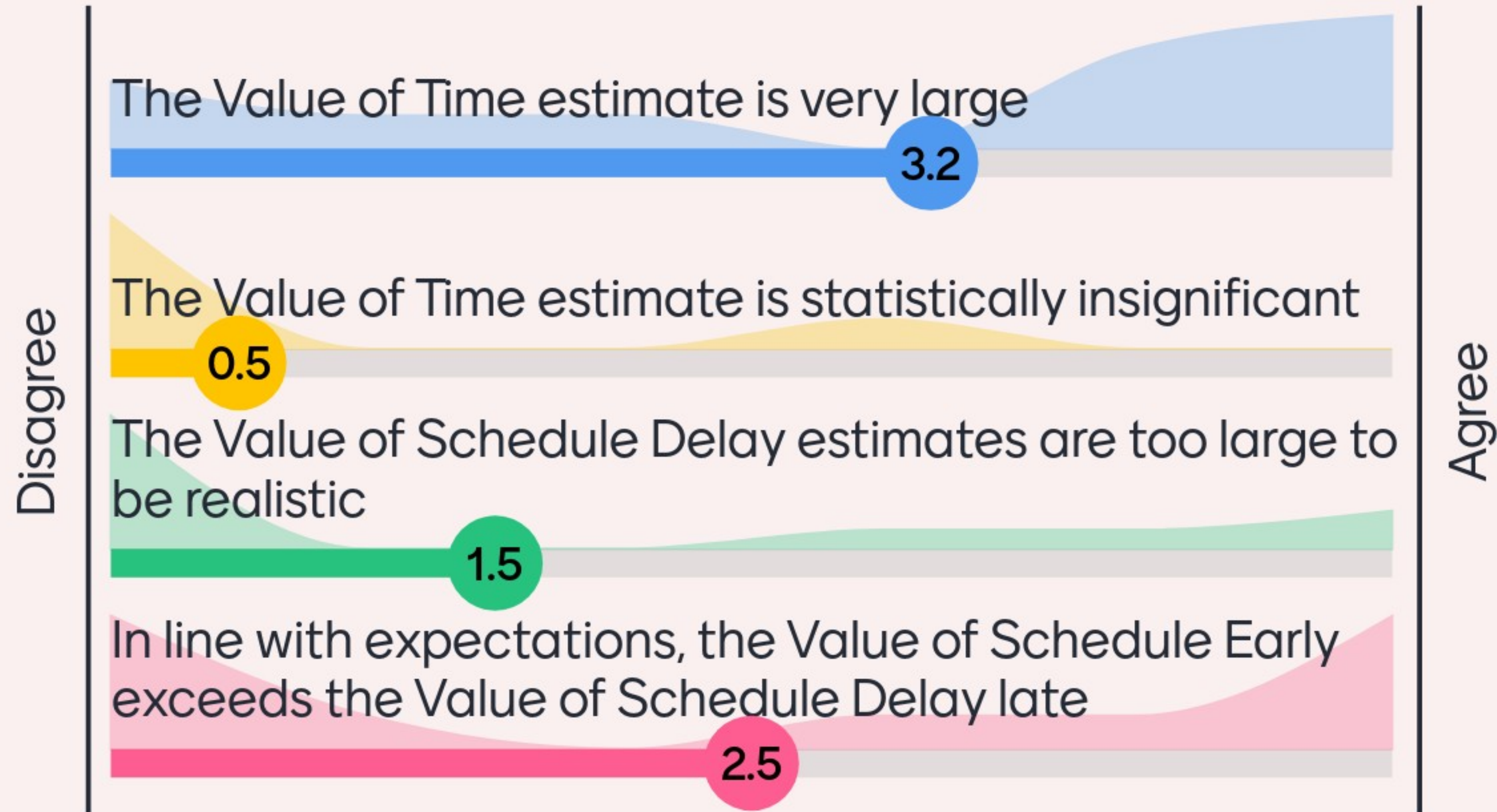
- **Value of schedule delay early (VSDE):**

$$-\beta^R \Delta R_{icj} = \beta^{SDE} \Delta SDE_{icj} \rightarrow \Delta SDE_{icj} = -1 \rightarrow -\Delta R_{icj} = -\frac{\beta^{SDE}}{\beta^R}$$

- **Value of time (VSDL):**

$$-\beta^R \Delta R_{icj} = \beta^{SDL} \Delta SDL_{icj} \rightarrow \Delta SDL_{icj} = -1 \rightarrow -\Delta R_{icj} = -\frac{\beta^{SDL}}{\beta^R}$$

# What is your interpretation of the results?



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- **Results (*s.e.'s between parentheses*)**

VOT	€ 35.05 (€ 4.158)
VSDE	€ 23.22 (€ 2.211)
VSDL	€ 17.16 (€ 1.621)

- **Willingness to pay estimates are high**
  - People are more sensitive to tolls
  - Relatively high share of high income households
- **VSDE > VSDL?**
  - Constraints in the morning rather than at work

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# Discrete choice (3)

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# Discrete choice (3)

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- **How to estimate these types of models?**

- **Overview**

	# Alternatives	Coefficients
1. Binary Logit	2	Homogeneous
2. Multinomial Logit with alternative specific parameters	>2, <~10	Differ between alternatives
3. Nested Logit	>2, <~10	Usually homogeneous
4. Conditional Logit	>2	Homogeneous

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- **Recall:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}}$$

**But now let the coefficients be alternative-specific:**

$$\Pr(Y = A) = \frac{e^{\beta_A x_A}}{e^{\beta_A x_A} + e^{\beta_B x_B} + e^{\beta_C x_C}}$$

- **We cannot identify all the coefficients  $\beta_A, \beta_B, \beta_C$ , because we compare the results to a reference category**
  - » **Think of dummies**
- **Illustration: we can write the probability only in terms of differences with respect to one reference category, e.g.:**

$$\Pr(Y = A) = \frac{1}{1 + e^{\beta_B x_B - \beta_A x_A} + e^{\beta_C x_C - \beta_A x_A}}$$

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- All the coefficients are compared to one base category!
- Coefficients are different for different alternatives
- Particularly useful when outcomes do not have a logical ordering
  - Bus, car, train
  - Holiday destinations
  - Otherwise: OLS or Ordered Logit
- If the number of alternatives is very large → too many coefficients to interpret meaningfully

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- **Independence of irrelevant alternatives**
  - Adding an alternative does not affect the relative odds between two other options considered
  - **Solution: use Nested Logit**
    - Allows for correlation within nests
  
- **Software**
  - NLOGIT in STATA
  - Use Biogeme software
  - Limdep/nlogit

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- Often, the number of alternatives is very large
  - Location choice
  - Route choice
  - Holiday destinations
  - Choice of car
  - Partner choice
  - ...
- With Multinomial Logit this becomes infeasible
  - Unique coefficients for each alternative
  - Not necessary for large choice sets

- Conditional Logit:

$$\Pr(d_j = 1) = \frac{e^{\beta' x_j}}{\sum_{k=1}^J e^{\beta' x_k}}$$

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- **How to deal with large choice sets?**
    - **Number of observations in your regressions is *number of alternatives*  $\times$  *respondents***
1. **Model aggregate choices**
  2. **Random selection of alternatives**
  3. **Estimate count data models (Poisson)**

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## 1. Model aggregate choices

### ▪ Modelling location choice

- Focus on aggregate areas (*e.g.* municipalities)

### ▪ Choice of cars

- Only distinguish between brands

- However, lack of detail makes results less credible



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## 2. Random selection of alternatives

### ▪ McFadden (1978)

- Choose a random subset of  $J$  alternatives for each choice set, including the chosen option
- This should not affect the *consistency* of the estimated parameters
- Small-sample properties are yet unclear

### ▪ How large should $J$ be?

- Applied in many good papers
  - e.g. Bayer et al. (2007, *JPE*)

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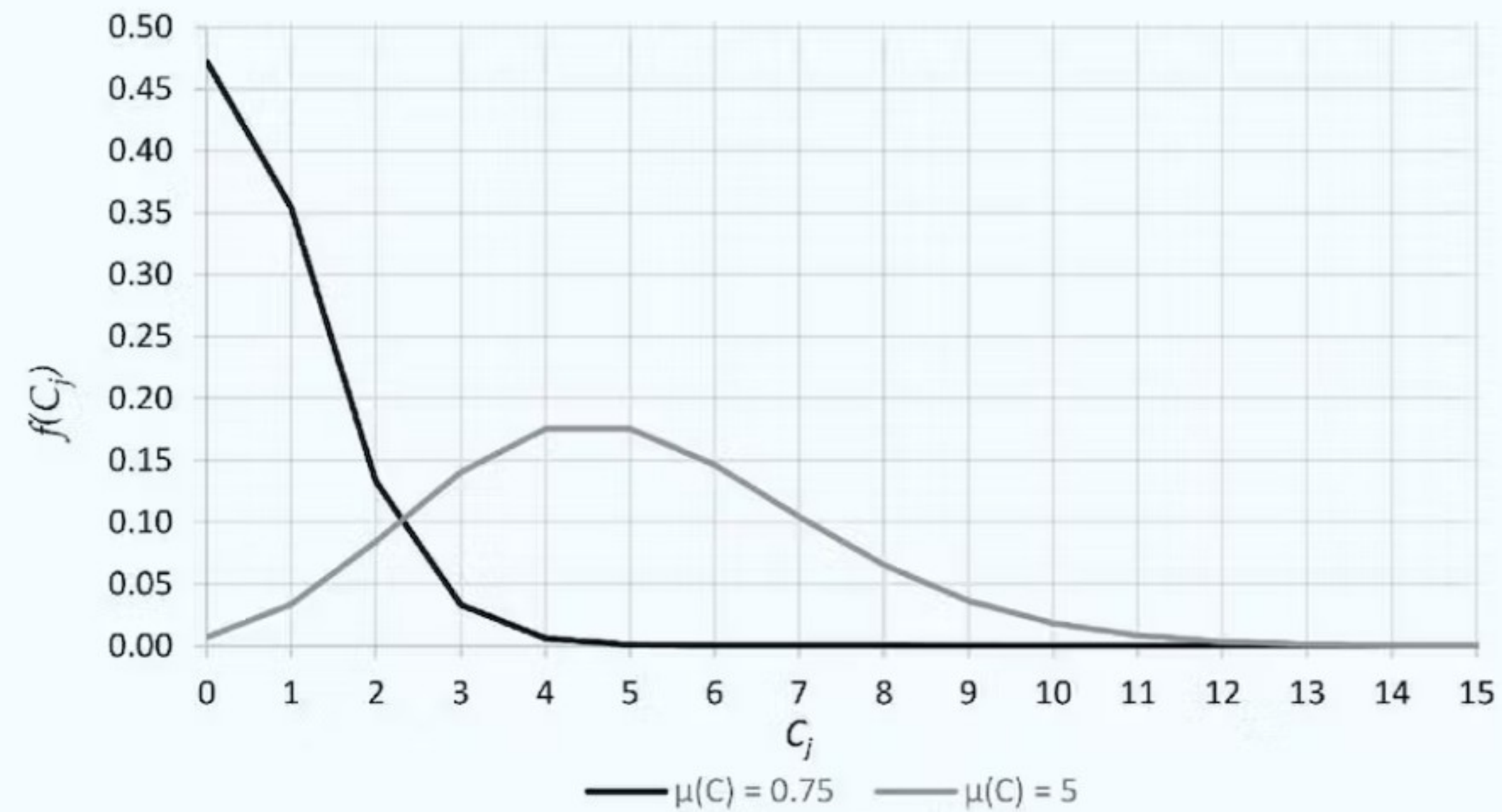
### 3. Estimate count data models

- **Estimate Conditional Logit by means of a Poisson model**
  
- **A Poisson regression is a count data model**
  - **Dependent variable is integer**
  - **... and should be Poisson distributed**

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### 3. Estimate count data models

- **Example of a Poisson distribution**



- **Equidispersion:  $\bar{y} = \sigma_y$**

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### 3. Estimate count data models

- **Estimate Conditional Logit by means of a Poisson model**

- **A Poisson regression is a count data model**

- **Dependent variable is integer**
- **... and should be Poisson distributed**
- $C_j = e^{\beta' x_j} + \epsilon$

where  $C_j$  is the # of decision makers that have chosen a certain alternative

- **Convenient interpretation of  $\beta$**

- **When  $x_j$  increases with one,  $C_j$  increases with  $\beta \times 100$  percent**

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### 3. Estimate count data models

- **A Poisson model should give identical parameters to the Conditional Logit**
  - **Maximum likelihood functions are identical *up to a constant***
  - **Guimarães et al. (2003)**
  
- **Hence, group observations based on their chosen alternatives**
  - **... the number of firms choosing a certain location**
  - **... the number of people buying a certain car**

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### 3. Estimate count data models

#### ▪ Implications

- You cannot include characteristics of the decision maker (*because you sum over decision makers*)!
- Homogeneous parameters across the population

#### ▪ Extensions

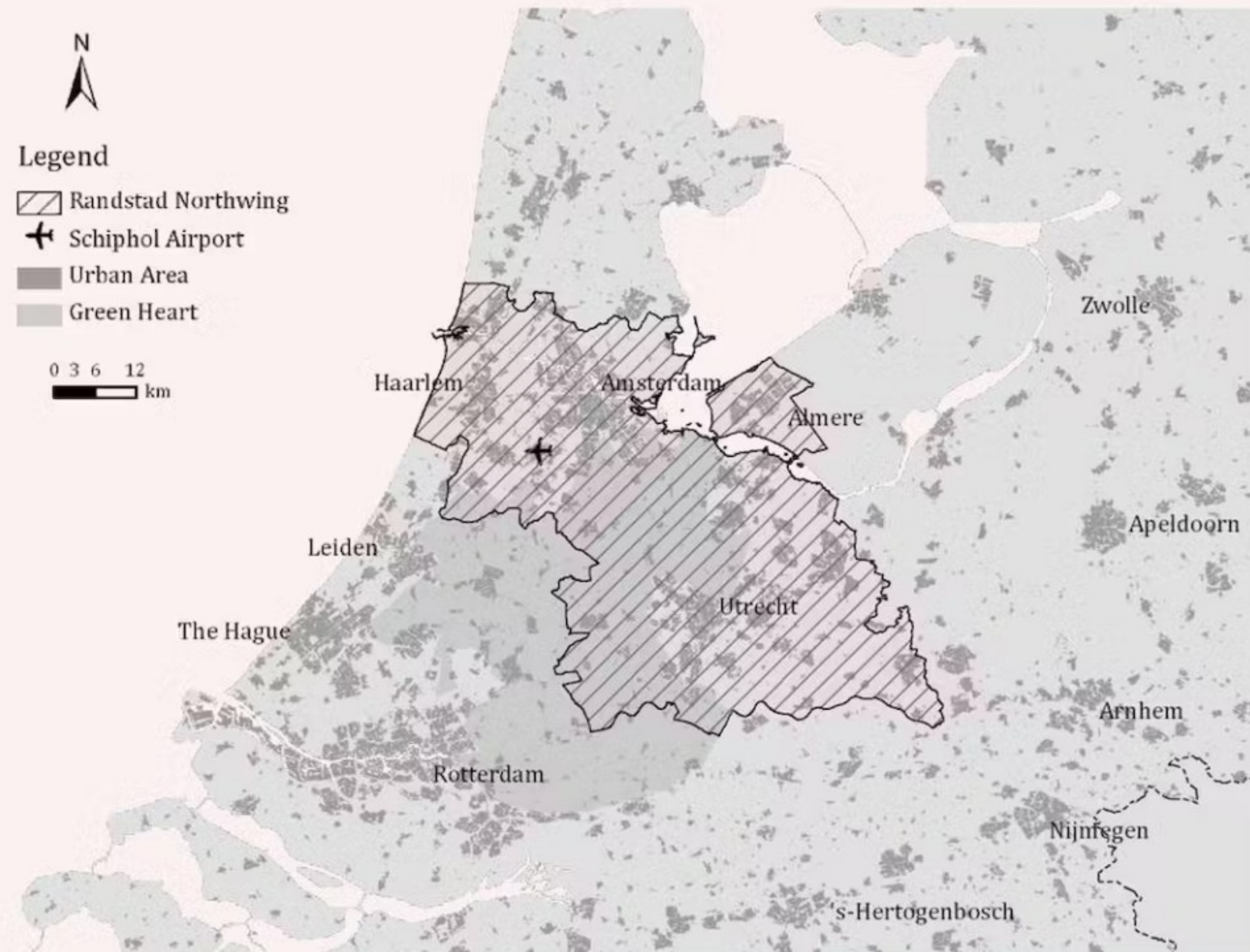
- Include fixed effects
- Negative binomial regression
- Zero-inflated models
- See Guimarães et al. (2004) for details

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- **Jacobs *et al.* (2013)**
  - Analyse location choices of business start-ups
  - Investigate the impact of *multinationals* on the number of business start-ups
  - In the Randstad Northwing
  
- **Multinationals may generate:**
  - Knowledge spillovers
  - Spin-offs
  - Potential customers (*output sharing*)

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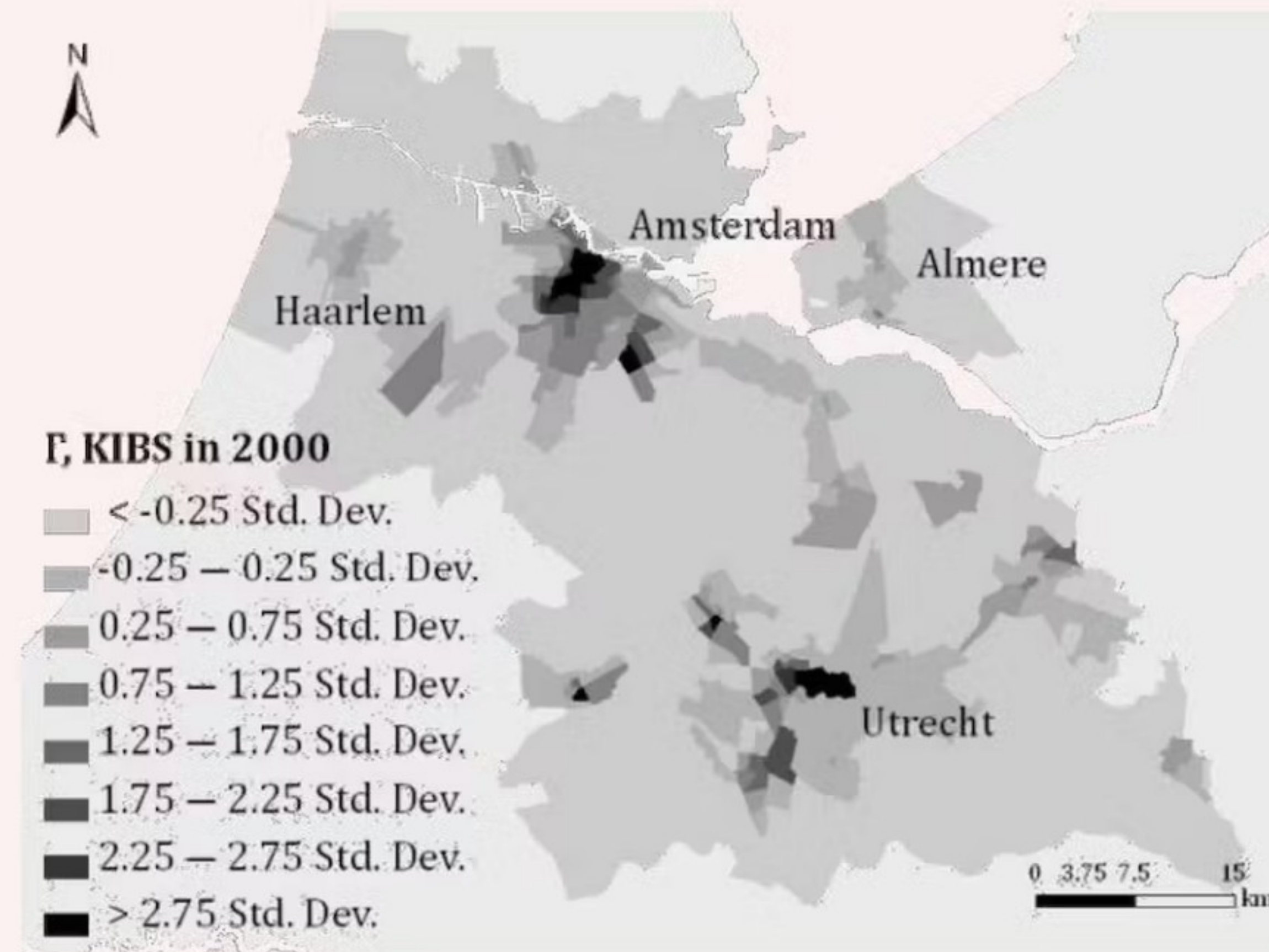
## ■ The Randstad Northwing





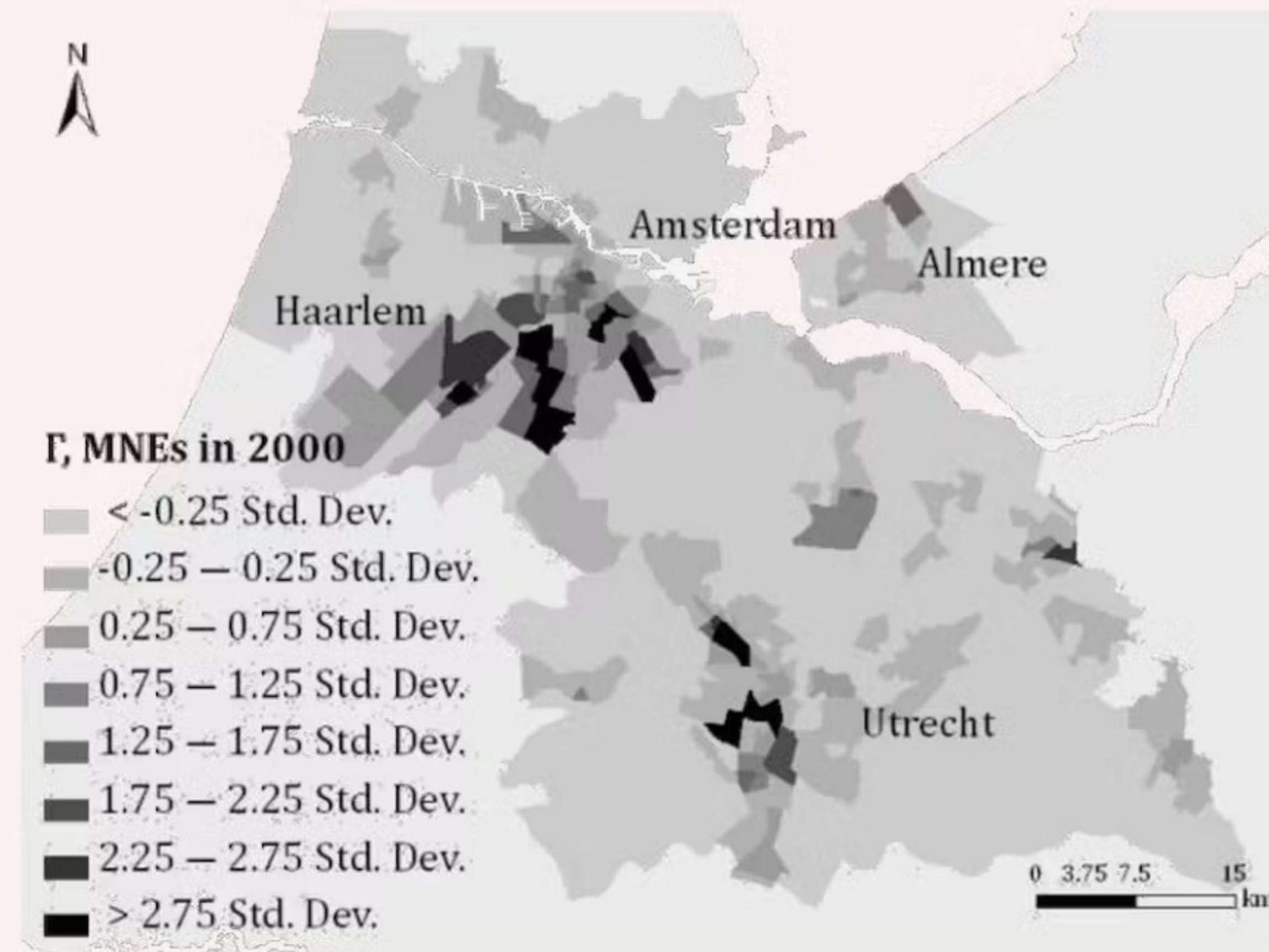
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## ■ Business services in the Randstad Northwing



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## ■ Multinationals in the Randstad Northwing



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- **Location choice model:**

$$\pi_{ij} = \alpha + \beta e_j^{MNE} + \gamma e_j^{BSF} + \delta e_j^{OF} + \zeta X_j + \eta_{j \in M} + \epsilon_{ij}$$

$i$	<b>firm</b>
$j$	<b>PC6 location (alternatives)</b>
$e_j^{MNE}$	<b>multinational employment</b>
$e_j^{BSF}$	<b>business services employment</b>
$e_j^{OF}$	<b>other employment</b>
$X_j$	<b>control variables</b>
$\eta_{j \in M}$	<b>municipality fixed effects</b>

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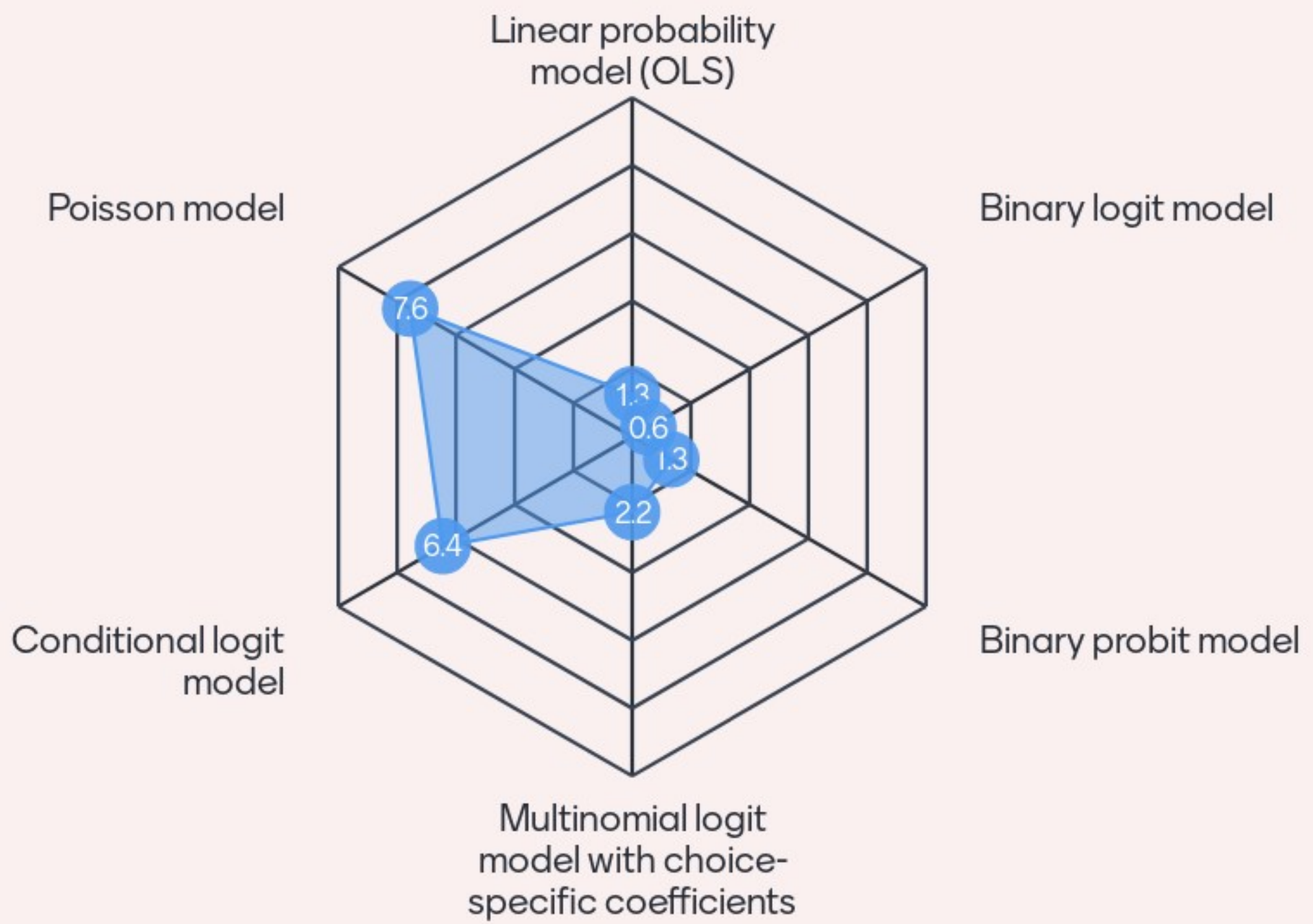
- **Probability  $\Pi$  that  $i$  chooses  $k$ :**

$$\Pr(d_j = 1) = \frac{e^{\alpha + \beta e_j^{MNE} + \gamma e_j^{BSF} + \delta e_j^{OF} + \zeta X_j + \eta_{j \in M}}}{\sum_{k=1}^J e^{\alpha + \beta e_k^{MNE} + \gamma e_k^{BSF} + \delta e_k^{OF} + \zeta X_k + \eta_{k \in M}}}$$

- **There are 13,655 locations**

→ **How would you estimate this model?**

# What regression method would you use to estimate this model?



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- **Problem: many locations**
  - **Use count data to estimate this model**
    - **There are no individual firm characteristics**
  - **Dependent variable: # start-ups per location**

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## ■ Results

Table – A POISSON MODEL

(Dependent variable: The number of business services start-ups per location)

	(1)	(2)	(3)
Multi-national employment density ( <i>log</i> )	0.0709*** (0.0151)	0.0422*** (0.0092)	0.0772*** (0.0121)
Business services employment density ( <i>log</i> )	0.4304*** (0.0240)	0.4374*** (0.0162)	0.3821*** (0.0214)
Other employment density ( <i>log</i> )	-0.2242*** (0.0162)	-0.2203*** 0.0071	-0.1352*** (0.0178)
Control variables (9)	No	Yes	Yes
Municipality fixed effects (61)	No	No	Yes
Number of locations	13,655	13,655	13,655
Log-likelihood	-13,146.903	-13,051.163	-12,709.249

Notes: We include locations with at least 10 employees in 2000. The coefficients can be interpreted as elasticities and differ from Jacobs et al. (2013) because of a slightly different set of controls and because we estimate Poisson models instead of Negative-binomial regressions. Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.10$ .

→ Please interpret the results in column (3)

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- **Multinationals attract new business services**
  - **The effect of other business services on start-ups, is however, much larger**
  
- **Coefficients are convenient to interpret**
  - *e.g.* a 1% increase in multinational empl. leads to an increase of start-ups of 0.077% (in Column (1))



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## Types of data

- Revealed preference (RP) data
  - Observed or reported actual behaviour
  
- Stated preference (SP) data
  - Respondents are confronted with hypothetical choice sets
  
- Combinations of RP and SP

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## Advantages of RP data

- **Based on actual behaviour!!**
- **Use existing (large) data sources**
  - **Cheaper**
  - **No expensive experiments**
- **Panels of the same individuals over a long time**

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## Disadvantages of RP data

- **Lack of variability**
- **Collinearity (e.g. price and travel times)**
- **Lack of knowledge on the choice set**
- **Not possible with new choice alternatives**
- **Actual behaviour may not be first choice**
  - **University numerus fixus**
- **Perception errors and imperfect information**
  - **Airline tickets**

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- **Example of stated preference question**
  - **Different from contingent valuation!**

**Suppose you have to ship a product from A to B**

<b>Option 1</b>		<b>Option 2</b>	
Price:	€ 1,000	Price:	€ 750
Handling time:	3 days	Handling time:	1 week
% does not arrive:	1.0%	% does not arrive:	1.3%
What alternative will you choose?			

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## Advantages of SP data

- **New alternatives**
- **New attributes**
- **Large variability is possible**
- **Problems of collinearity can be solved**
  - **'Orthogonal design'**
- **Choice set is clearly defined**

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## Disadvantages of SP data

- Information bias
  - The respondent has incorrect information on the context
  - Make your experiment as realistic as possible
  
- Starting point bias
  - Respondents are influenced by the set of available responses to the experiment
  - Test your design and choose realistic attribute values

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## Disadvantages of SP data

- Hypothetical bias
  - Individuals tend to respond differently to hypothetical scenarios than they do to the same scenarios in the real world.
  - Cognitive incongruity with actual behaviour
  - Again: make your experiment as realistic as possible
  - But otherwise hard to mitigate...
  
- Strategic bias
  - Respondent wants a specific outcome
  - (S)he fills in answers that are in line with desired outcomes

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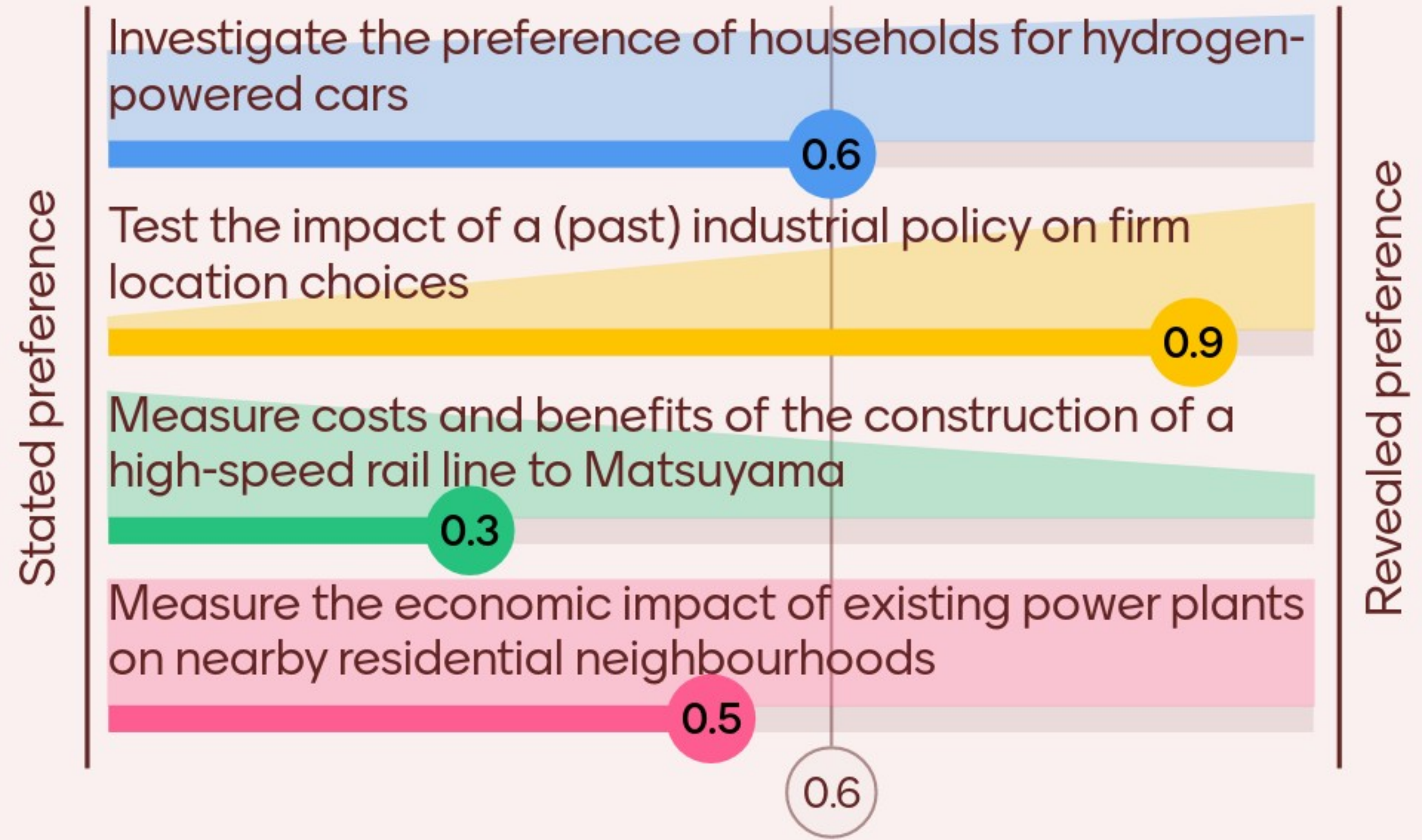
## Disadvantages of SP data

- Unintentional biases
  - Information, starting point, hypothetical bias
- Intentional biases
  - Strategic bias
- Errors
  - Boredom
  - Respondents do not carefully read instructions
  - Respondents do not understand the questions

If there is good data available, I would prefer RP  
*(personal opinion)*



# Would you use SP or RP in the following cases:



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## Discrete choice:

- **Random utility framework**
- **Generalisations of logit models**
  - LPM ( $J = 2$ )
  - Binary logit/probit ( $J = 2$ )
  - Multinomial logit ( $2 < J < 10$ )
  - Nested logit ( $2 < J < 10$ )
  - Conditional logit ( $J > 2$ )
- **Conditional Logit models can be estimated by count data models**
  - Cannot include characteristics of the decision maker
- **Data**
  - Stated preference or revealed preference data

# Discrete choice (3)

Applied Econometrics for Spatial Economics

**Hans Koster**

*Professor of Urban Economics and Real Estate*