Discrete choice (1)

Applied Econometrics for Spatial Economics

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1. Introduction

- **Today:**
  1. Spatial econometrics
  2. Discrete choice
  3. Identification

- **Tomorrow:**
  4. Hedonic pricing
  5. Quantitative spatial economics

1. Introduction

- **Today:**
  1. Spatial econometrics
  2. Discrete choice
     - Random utility framework, estimating binary and multinomial regression models
  3. Identification

- **Tomorrow:**
  4. Hedonic pricing
  5. Quantitative spatial economics

1. Introduction

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1. Introduction

- *Continuous* choice: estimate marginal changes in behaviour
  - E.g. "when fuel price increases by 10%, the demand for fuel will decrease by 2%"
  - Standard micro-economic theory applies

- Transport demand often has a discrete (binary) nature
  - Some x impacts a discrete y
  - Then use discrete choice methods

1. Introduction

- **Discrete choice methods**
  - ***Dependent* variable y_i is discrete**

- **Why not use OLS?**

- **Let's have the standard OLS equation**

$$y_i = \beta x_i + \epsilon_i \tag{1}$$

where i indexes the individual

1. Introduction

- OLS may be consistent for binary choice
  - *But,* y_i *(and therefore* ϵ) is *not* normally distributed

- Horrace and Oaxaca (2006)
  - Leads to biased and inconsistent estimates if $\hat{y}_i$ lies 'often' outside the [0,1] interval
  - I show later today why that is an issue…

- OLS does not necessarily provides a link with economic theory

- Not suitable for multinomial choice

2. The random utility framework

- **Indirect utility may be given by:**

  $$U_{iA} = V_A(travel\ time_A) + \epsilon_{iA} \quad (2)$$

  $$U_{iB} = V_B(travel\ time_B) + \epsilon_{iB} \quad (3)$$

  - V_A, V_B ➔ **deterministic utility**

- **Random terms:** $\epsilon_{iA}, \epsilon_{iB}$**:** **random taste variation**
  - **Random utility model (RUM)**
  - **Note that the levels of** U_{iA} **and** U_{iB} **are not directly observed!**

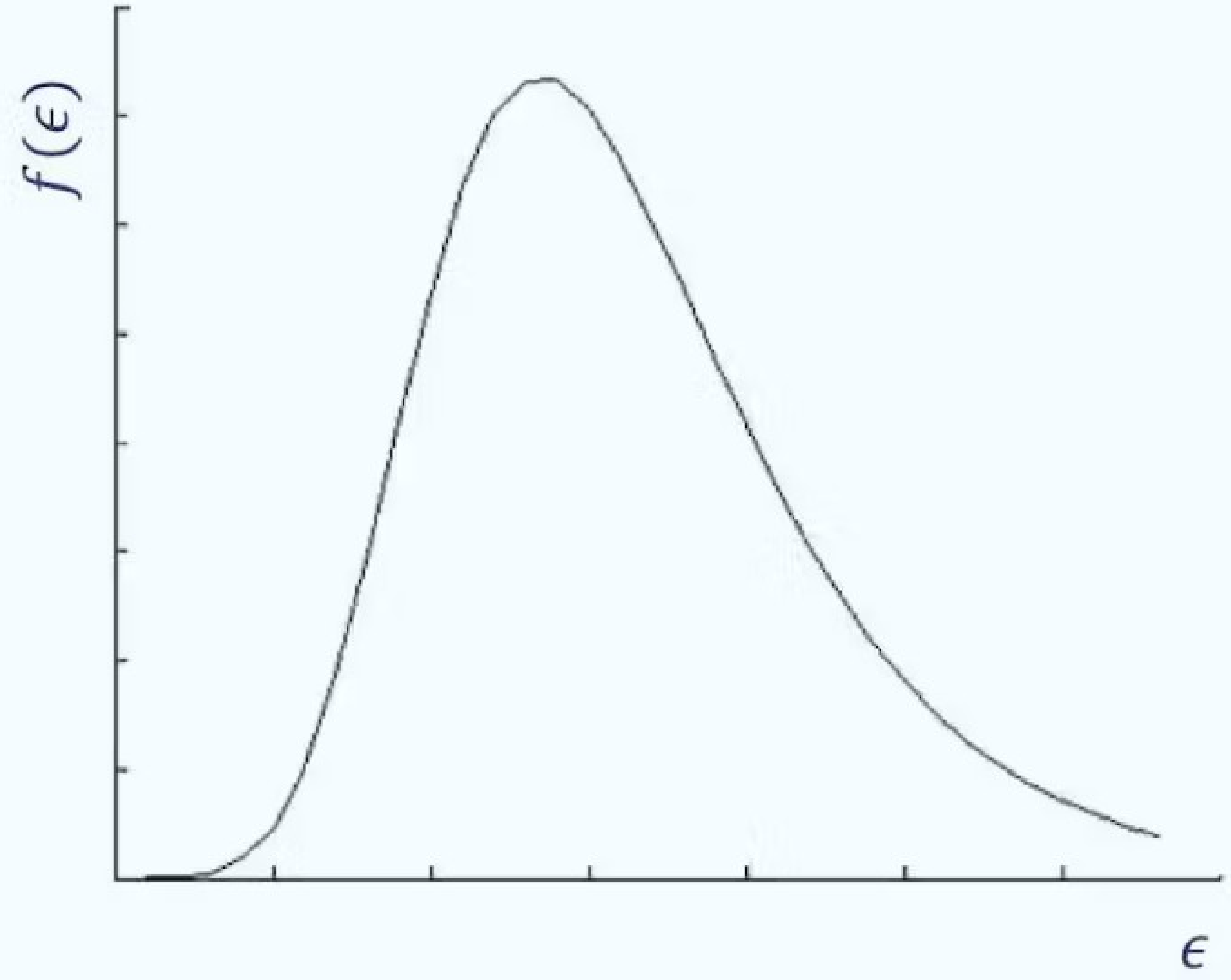
- $\Pr(Y = A) = \Pr(U_{iA} > U_{iB})$
- $\Pr(V_A + \epsilon_{iA} > V_B + \epsilon_{iB}) = \Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$

2. The random utility framework

- Two things are unknown
  - Which distribution for ϵ's?
  - What is the functional form for V_A and V_B?

2. The random utility framework

- **Which distribution for ϵ's?**
  - ϵ's are unobserved
  - You draw them from a distribution
  - Logit: Extreme Value Type I distribution



2. The random utility framework

- **Which distribution for ϵ's?**
  - **Extreme Value Type I distribution**
  - **Generates simple closed-form solutions!**

    $\rightarrow \quad \Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$
  - **Daniel McFadden (1964)**

2. Random utility framework

- **It appears that:**

$$\Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} \tag{4}$$

- **With two alternatives this can be written as:**

$$\Pr(Y = A) = \frac{1}{1 + e^{V_B - V_A}}$$

2. The random utility framework

- Which functional form for V_A and V_B?
  - Can be any function
  - Linear function is often assumed
  - Can be extended with multiple variables

    $$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \epsilon_{jA} \quad (5)$$

    $$U_{jB} = \beta p_{jB} + \kappa t_{jB} + \epsilon_{jB} \quad (6)$$

    where p_{jA} is the price of a trip and t_{jA} is travel time of alternative j
  - $\beta < 0, \kappa < 0$

- Recall (from previous slide):
  - $\Pr(Y = A) = \frac{1}{1+e^{\beta(p_{jB}-p_{jA})+\kappa(t_{jB}-t_{jA})}}$

3. Value of time

- Important concept in Transport Economics: *Value of Time* (VOT)
  - "How much are you willing to pay to reduce your travel time with one hour, *holding utility constant*"

- Let's take the deterministic utility function

$$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \varepsilon_{jA} \qquad (7)$$

- When t_{jA} is measured in hours, the VOT can be written as κ/β

3. Value of time

- Value of time is often used in cost benefit analyses

- VOT depends on trip purpose
  - Business €26.25/h
  - Commuting €9.25/h
  - Social purpose €7.50/h

- VOT depends on income
  - About 50% of net income

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3. Value of time

- **Other applications**
  - **Value of a Statistical Life (VSL)**
    The VSL is the local tradeoff between fatality risk and money
  - **Value of schedule delay (VSD)**
    The VSD is the local tradeoff between being too early/late and money
  - **Etc.**
  - **... What is necessary is a cost/reward parameter in the discrete choice experiment**

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4. Multiple alternatives

- **The choice probability for two alternatives:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B}}$$

- **Usually there are <u>more alternatives in the choice set</u>**
  - **Train, bus, car**
  - **Rotterdam, Antwerp, Hamburg**
  - **Routes to the VU**

- **Simply extend the logit formula:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}}$$

4. Multiple alternatives

- **The aggregate utility derived from the choice set is summarised by the logsum:**

  $$\mathrm{E[CS]} = \frac{1}{\upsilon}\ln\left(\mathrm{e}^{\beta x_A} + \mathrm{e}^{\beta x_B} + \mathrm{e}^{\beta x_C}\right)$$

  - υ **is the marginal utility of income**
  - **Can be used in welfare estimates**

- **Assume** $\beta x_A = \beta x_B = 10$

- **Now alternative** C **is added and** $\beta x_C = 1$

- **The average utility per alternative decreases from** 10 **to** 7 **but** $\mathrm{E[CS]}$ **increase**
  - **'Love of variety' effect**

4. Multiple alternatives

- Property of logit formula:
  - The *ratios* of choice probabilities for A and B do not depend on whether or not C is in the choice set
  - Independence of irrelevant alternatives
  - $$\frac{\Pr(Y=A)}{\Pr(Y=B)} = \frac{\left(\frac{e^{\beta x_A}}{e^{\beta x_A}+e^{\beta x_B}+e^{\beta x_C}}\right)}{\left(\frac{e^{\beta x_B}}{e^{\beta x_A}+e^{\beta x_B}+e^{\beta x_C}}\right)} = \frac{e^{\beta x_A}}{e^{\beta x_B}}$$
  - Let's find out whether this is a desirable property...

4. Multiple alternatives

- 'The *Red Bus-Blue Bus*' problem

- Choice set 1: Train, red bus, blue bus
- Assume market shares are 70, 15 and 15%

	Train	Red bus	Blue bus
V	2.54	1	1
Prob	0.700	0.150	0.150

- Choice set 2: Train, red bus, so:

	Train	Red bus
V	2.54	1
Prob	0.823	0.177

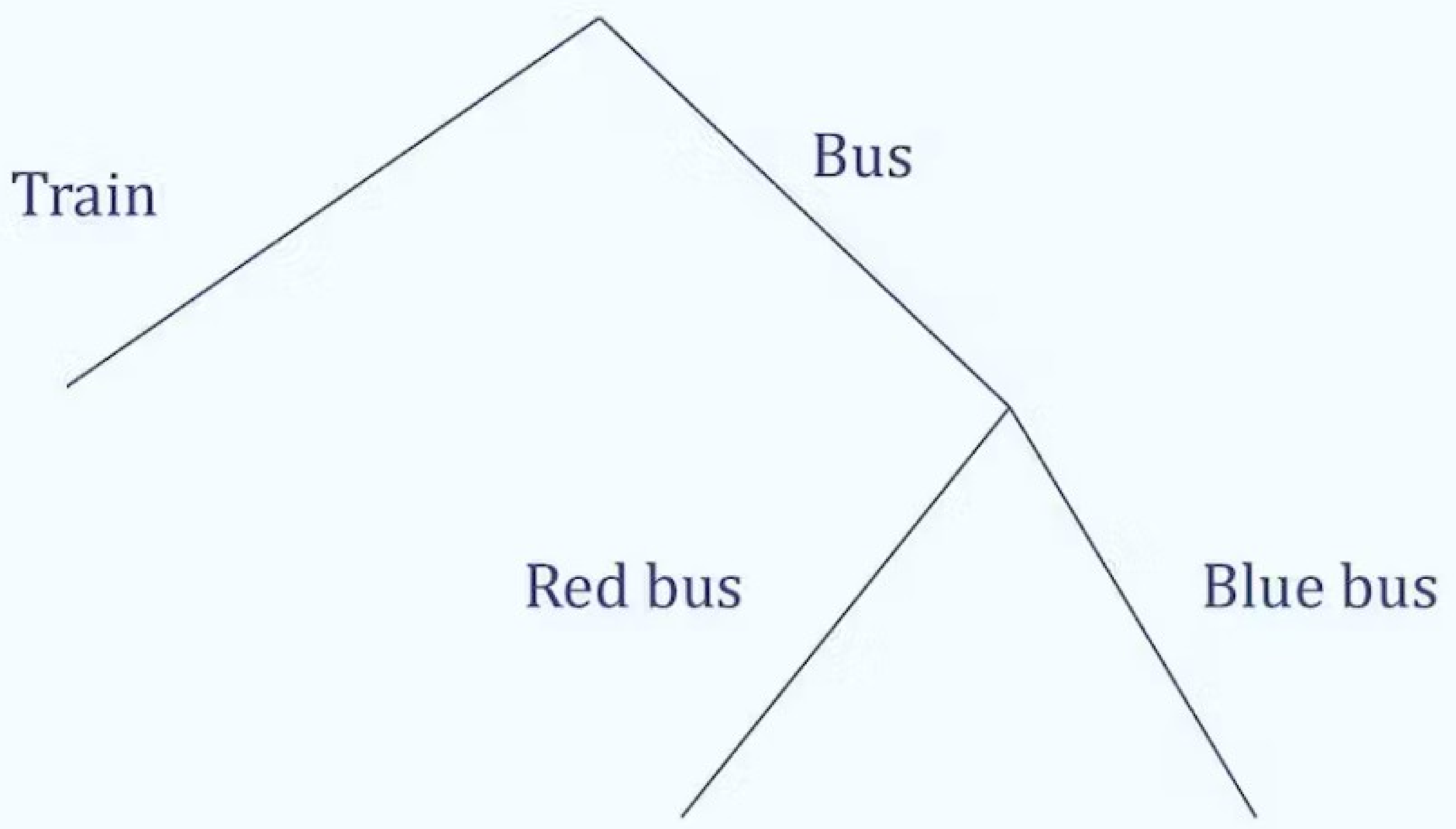
- Probability to take the bus in choice set 2 is

$$\frac{e^1}{e^{2.54}+e^1} = 0.177$$

  - Higher probability – not very realistic as red buses and blue buses are identical

4. Multiple alternatives

- So, when some alternatives are more similar than other alternatives, the use of multinomial choice model may be misleading
- Use nested logit!



4. Multiple alternatives

- **Nested logit takes into account correlation between alternatives**
    - **But define nests yourself!**

2. The random utility framework

- **Let us define utility as follows:**

  $U_{jg} = V_j + W_g + \epsilon_{jg}$

  V_j **only differs within nests between alternatives** j

  W_g **only differs between nests** g

2. The random utility framework

- **We may write the probability to choose an alternative:**
  - $\Pr(d_j = 1) = \Pr(g) \cdot \Pr(j \mid g)$
  - $\Pr(j \mid g) = \dfrac{e^{V_j/\lambda_g}}{\sum_{k \in g} e^{V_k/\lambda_g}}$
  - $\Pr(g) = \dfrac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}}$

  with $I_g = \log\left(\sum_{j \in g} e^{V_j/\lambda_g}\right)$

- $\lambda_g = 1 \Rightarrow$ **no correlation (multinomial logit)**
- $\lambda_g \to 0 \Rightarrow$ **perfect correlation (red bus/blue bus)**

2. The random utility framework

- **When j and k are in the same nest:**

$$\frac{\Pr\left(d_j = 1\right)}{\Pr(d_k = 1)} = \frac{e^{W_g + V_j / \lambda_g}}{e^{W_g + V_k / \lambda_g}} = \frac{e^{W_g + V_j}}{e^{W_g + V_k}} = \frac{e^{V_j}}{e^{V_k}}$$

  - **IIA property holds *within* nests**

- **When $\lambda_g \to 0$:**
  - $\Pr(j \mid g) = \dfrac{e^{V_j / \lambda_g}}{\sum_{k \in g} e^{V_k / \lambda_g}} = 0.5$
  - $\Pr(g) = \dfrac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}} = \dfrac{e^{W_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}}}}$
  - **Hence, multinomial logit *between* nests**

2. The random utility framework

- So, nested logit probability depends on
  - Probability to choose a nest
  - Probability to choose an alternative within the nest

- Note that Nested Logit does not imply a *sequential* choice

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Discrete choice (2)

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1. Introduction

- How to estimate binary discrete choice models?
- Three main options
  1. Linear probability model
  2. Logit
  3. Probit

2. Linear probability model

Advantages:

- **Consistent when** $0 \leq \hat{y}_j \leq 1 \; \forall \; j$

- **Easy to interpret**
  - $\frac{\partial \Pr(d_j = 1)}{\partial x} = \beta$

- **Computationally feasible**
  - **Important for large panel datasets**

- **In practice, leads to very similar results as Logit and Probit**

2. Linear probability model

Disadvantages:

- No direct link with structural parameters of utility function
  - e.g. not able to calculate aggregate utility from choice set
- Biased for small samples and possibly inconsistent marginal effects
  - Linearity?
- Not suitable for multinomial choices

3. Logit

- **Let's define**

$$\Pr(d_j = 1) = \frac{1}{1+e^{-\beta' x_j}}$$

- **Example: regress 0/1 variable on *differences* in characteristics of the alternatives**

Chosen_B	$\text{Price}_B\text{-Price}_A$	$\text{Time}_B\text{-Time}_A$
1	-14	5
0	5	0
0	15	-20
1	-8	13
1	-10	3
1	3	-5
0	20	10

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3. Logit

- **The change in the probability for one unit increase in x**
  - $$\frac{\partial \Pr(d_j=1)}{\partial x_j} = \beta \frac{e^{-\beta' x_j}}{\left(1+e^{-\beta' x_j}\right)^2}$$

- **Marginal effect depends on x_j, so is not constant/linear**
  - **For example, evaluate at mean values of x**

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3. Logit

- **Software**
  - LOGIT **or** LOGISTIC **in STATA**
  - REGRESSION – BINARY LOGISTIC **in SPSS**

- **In STATA you can select to report marginal effects**
  - **Use** MARGINS **after** LOGIT **command**
  - **Choose at which x the values are evaluated (e.g. at means)**

3. Logit

Advantages of Logit:

- Predicted probability is always between one and zero
- Clear link to random utility framework
  - Log-sum may be used for welfare calculations
- Closed-form marginal effects
  - Usually leads to very similar results as Probit
- Can include 'fixed effects' (XTLOGIT in STATA)
  - *e.g. to* control for individual heterogeneity

Disadvantages of Logit:

- Why Extreme Value Type I distribution for ϵ?

- Maximum likelihood / non linear model

4. Probit

- We may also assume that ϵ_j is normally distributed, so $\epsilon_j = N(0, \sigma^2)$
  - This implies $\Pr(d_j = 1) = \Phi(\beta' x_j)$
  - However, no closed-form for cumulative normal distribution!

- Marginal effects:

$$\frac{\partial \Pr(d_j = 1)}{\partial x_j} = \beta \phi(\beta x_j)$$

where $\phi(\cdot)$ is the density function of the normal distribution

4. Probit

Advantages:

- Normal distribution for ϵ_j may seem more reasonable
- Probability is always between one and zero

Disadvantages:

- No closed-form marginal effects
- Hard to include many fixed effects

4. Probit

- How to choose between the three models?
  - Probit estimates ≈ Logit estimates
  - Check for robustness of marginal effects
  - Large sample and interested in marginal effects?
    - → Usually linear probability model!
    - → There is an ongoing debate in economics on this issue

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1. Introduction

- **How to estimate these types of models?**

- **Overview**

	# Alternatives	Coefficients
1. Binary Logit	2	Homogeneous
2. Multinomial Logit with alternative specific parameters	>2, <~10	Differ between alternatives
3. Nested Logit	>2, <~10	Usually homogeneous
4. Conditional Logit	>2	Homogeneous

2. Multinomial logit

- **Recall:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}}$$

But now let the coefficients be alternative-specific:

$$\Pr(Y = A) = \frac{e^{\beta_A x_A}}{e^{\beta_A x_A} + e^{\beta_B x_B} + e^{\beta_C x_C}}$$

- **We cannot identify all the coefficients β_A, β_B, β_C, because <u>we compare the results to a reference category</u>**
  - » **Think of dummies**

- **Illustration: we can write the probability only in terms of differences with respect to one reference category, e.g.:**

$$\Pr(Y = A) = \frac{1}{1 + e^{\beta_B x_B - \beta_A x_A} + e^{\beta_C x_C - \beta_A x_A}}$$

2. Multinomial logit

- All the coefficients are compared to one base category!
- Coefficients are different for different alternatives
- Particularly useful when outcomes do not have a logical ordering
  - Bus, car, train
  - Holiday destinations
  - Otherwise: OLS or Ordered Logit
- If the number of alternatives is very large ➔ too many coefficients to interpret meaningfully

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3. Nested logit

- Independence of irrelevant alternatives
  - Adding an alternative does not affect the relative odds between two other options considered
  - Solution: use Nested Logit
    → Allows for correlation within nests

- Software
  - NLOGIT in STATA
  - Use Biogeme software
  - Limdep/nlogit

4. Conditional logit

- Often, the number of alternatives is very large
  - Location choice
  - Route choice
  - Holiday destinations
  - Choice of car
  - Partner choice
  - ...

- With Multinomial Logit this becomes infeasible
  - Unique coefficients for each alternative
  - Not necessary for large choice sets

- Conditional Logit:

$$\Pr(d_j = 1) = \frac{e^{\beta' x_j}}{\sum_{k=1}^{J} e^{\beta' x_k}}$$

4. Conditional logit

- How to deal with large choice sets?
  - Number of observations in your regressions is *number of alternatives* $\times$ *respondents*

1. Model aggregate choices
2. Random selection of alternatives
3. Estimate count data models (Poisson)

1. Model aggregate choices

- Modelling location choice
  - Focus on aggregate areas (*e.g.* municipalities)

- Choice of cars
  - Only distinguish between brands

- However, lack of detail makes results less credible

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2. Random selection of alternatives

- McFadden (1978)
  - Choose a random subset of J alternatives for each choice set, including the chosen option
  - This should not affect the *consistency* of the estimated parameters
  - Small-sample properties are yet unclear

- How large should J be?

- Applied in many good papers
  - e.g. Bayer et al. (2007, *JPE*)

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4. Conditional logit

3. Estimate count data models

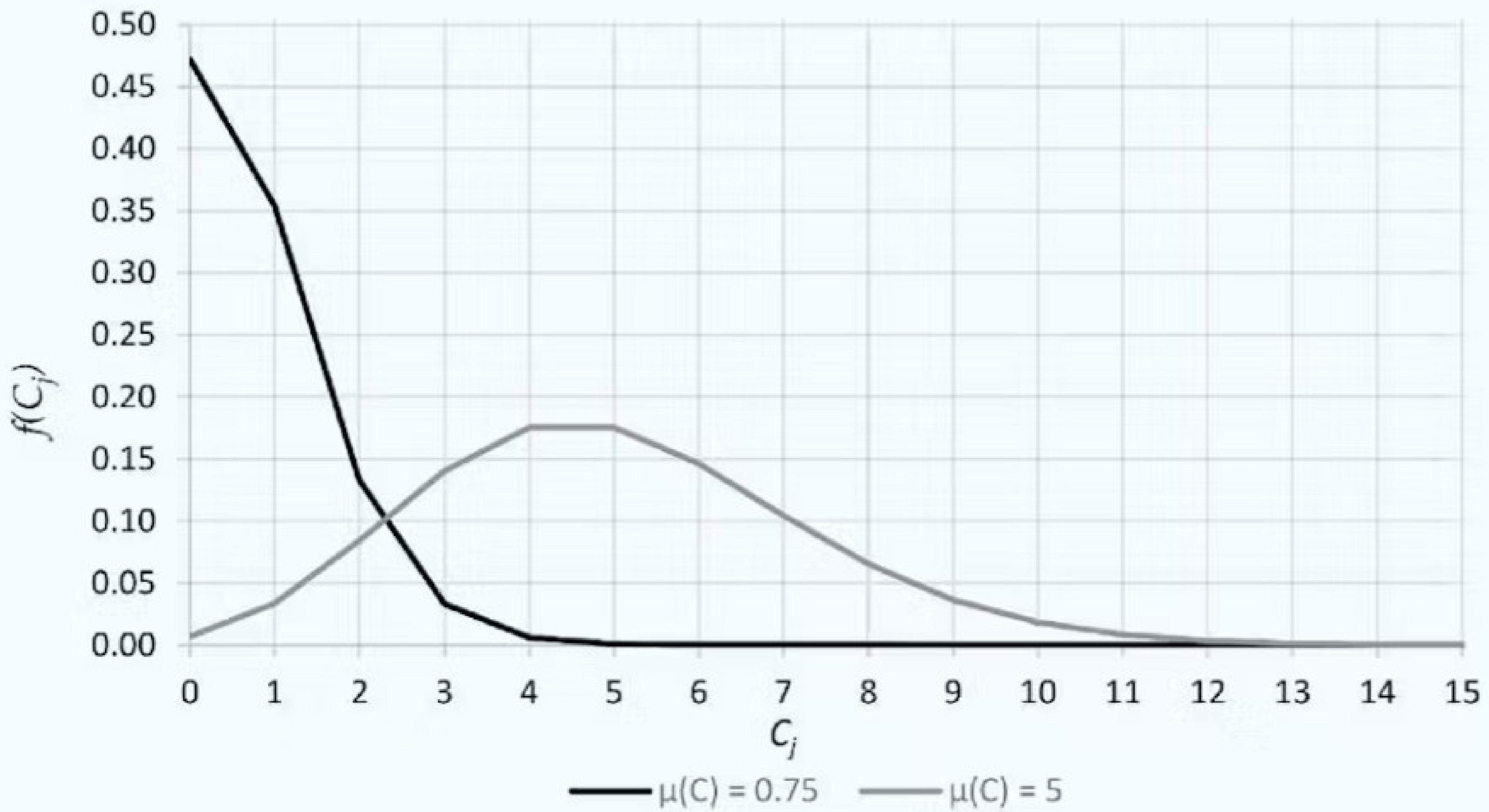
- Estimate Conditional Logit by means of a Poisson model

- A Poisson regression is a count data model
  - Dependent variable is integer
  - … and should be Poisson distributed

4. Conditional logit

3. Estimate count data models

- Example of a Poisson distribution



- Equidispersion: $\bar{y} = \sigma_y$

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3. Estimate count data models

- Estimate Conditional Logit by means of a Poisson model

- A Poisson regression is a count data model
  - Dependent variable is integer
  - ... and should be Poisson distributed
  - $C_j = e^{\beta' x_j} + \epsilon$
    where C_j is the # of decision makers that have chosen a certain alternative

- Convenient interpretation of β
  - When x_j increases with one, C_j increases with $\beta \times 100$ percent

3. Estimate count data models

- A Poisson model should give identical parameters to the Conditional Logit
  - Maximum likelihood functions are identical *up to a constant*
  - Guimarães et al. (2003)

- Hence, group observations based on their chosen alternatives
  - … the number of firms choosing a certain location
  - … the number of people buying a certain car

3. **Estimate count data models**

- **Implications**
  - **You cannot include characteristics of the decision maker *(because you sum over decision makers)*!**
  - **Homogeneous parameters across the population**

- **Extensions**
  - **Include fixed effects**
  - **Negative binomial regression**
  - **Zero-inflated models**
  - **See Guimarães et al. (2004) for details**

Types of data

- Revealed preference (RP) data
  - Observed or reported actual behaviour
- Stated preference (SP) data
  - Respondents are confronted with hypothetical choice sets
- Combinations of RP and SP

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Advantages of RP data

- Based on actual behaviour!!
- Use existing (large) data sources
  - Cheaper
  - No expensive experiments
- Panels of the same individuals over a long time

5. Revealed preference and stated preference data

Disadvantages of RP data

- Lack of variability
- Collinearity (e.g. price and travel times)
- Lack of knowledge on the choice set
- Not possible with new choice alternatives
- Actual behaviour may not be first choice
  - University numerus fixus
- Perception errors and imperfect information
  - Airline tickets

5. Revealed preference and stated preference data

- Example of stated preference question
  - Different from contingent valuation!

Suppose you have to ship a product from A to B	
Option 1	**Option 2**
Price: € 1,000	Price: € 750
Handling time: 3 days	Handling time: 1 week
% does not arrive: 1.0%	% does not arrive: 1.3%
What alternative will you choose?	

5. Revealed preference and stated preference data

Advantages of SP data

- New alternatives
- New attributes
- Large variability is possible
- Problems of collinearity can be solved
  - 'Orthogonal design'
- Choice set is clearly defined

Disadvantages of SP data

- Information bias
  - The respondent has incorrect information on the context
  - Make your experiment as realistic as possible

- Starting point bias
  - Respondents are influenced by the set of available responses to the experiment
  - Test your design and choose realistic attribute values

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Disadvantages of SP data

- Hypothetical bias
  - Individuals tend to respond differently to hypothetical scenarios than they do to the same scenarios in the real world.
  - Cognitive incongruity with actual behaviour
  - Again: make your experiment as realistic as possible
  - But otherwise hard to mitigate…

- Strategic bias
  - Respondent wants a specific outcome
  - (S)he fills in answers that are in line with desired outcomes

Disadvantages of SP data

- Unintentional biases
  - Information, starting point, hypothetical bias

- Intentional biases
  - Strategic bias

- Errors
  - Boredom
  - Respondents do not carefully read instructions
  - Respondents do not understand the questions

If there is good data available, I would prefer RP *(personal opinion)*

6. Summary

Discrete choice:

- Random utility framework

- Generalisations of logit models
  - LPM ($J = 2$)
  - Binary logit/probit ($J = 2$)
  - Multinomial logit ($2 < J < 10$)
  - Nested logit ($2 < J < 10$)
  - Conditional logit ($J > 2$)

- Conditional Logit models can be estimated by count data models
  - Cannot include characteristics of the decision maker

- Data
  - Stated preference or revealed preference data

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