

# Quantitative spatial economics

Applied Econometrics for Spatial Economics

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1. Introduction
2. Model set-up
3. Recursive estimation
4. Empirical evidence
5. Counterfactuals
6. Summary

- Yesterday:
  1. Spatial econometrics
  2. Discrete choice
  3. Identification
- Today:
  4. Hedonic pricing
  5. **Quantitative spatial economics**

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- Yesterday:
  1. Spatial econometrics
  2. Discrete choice
  3. Identification
- Today:
  - ~~4. Hedonic pricing~~
  - 5. Quantitative spatial economics**
    - **General equilibrium models in spatial economics**

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- Empirical urban economics was often a **'reduced-form'** field
  - Effect of policy on marginal changes in behaviour

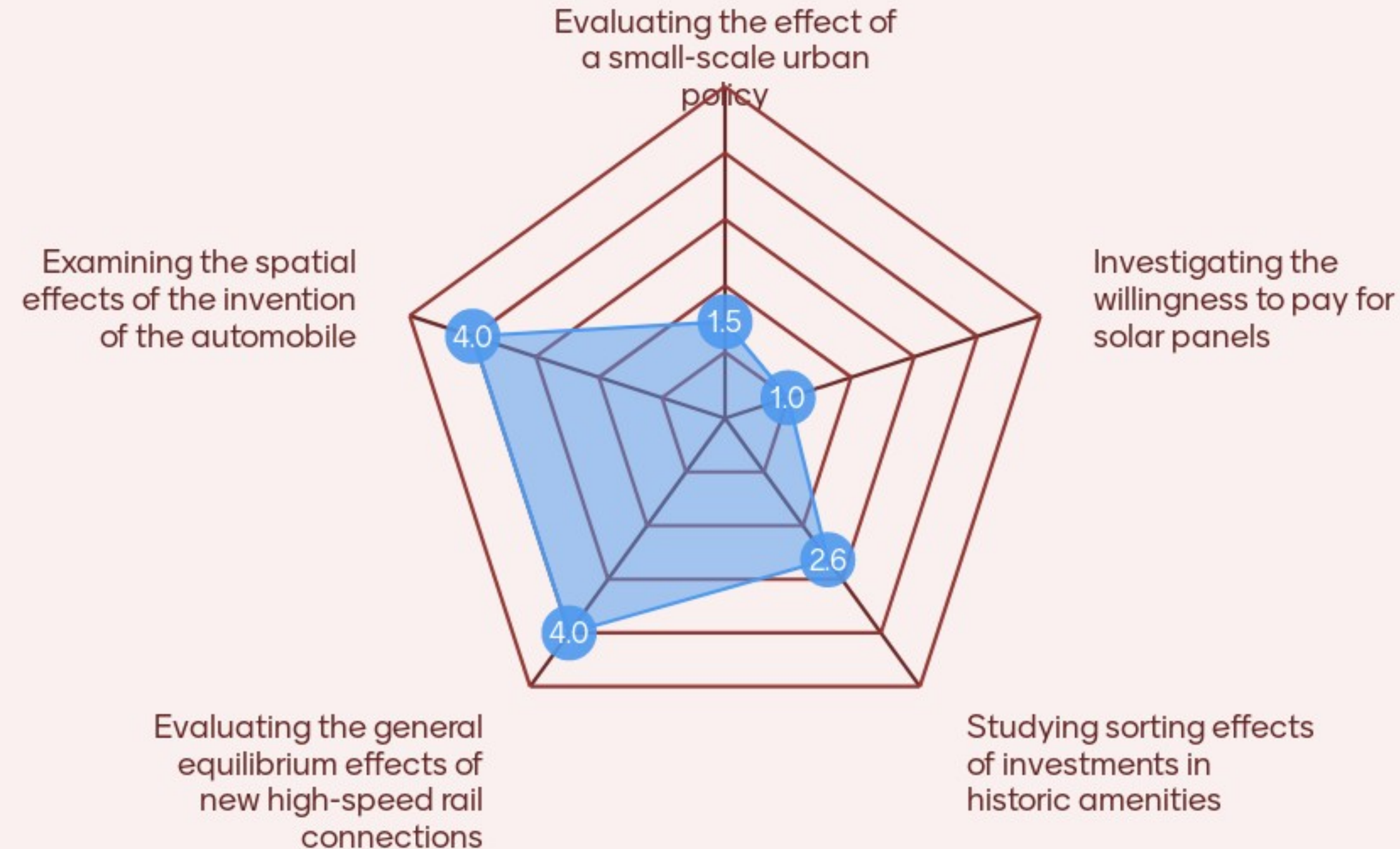
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- Empirical urban economics was often a **'reduced-form'** field
  - Effect of policy on marginal changes in behaviour
  
- Pros and cons
  - Few(er) assumptions (+)
  - Easy interpretation (no black box) (+)
  - Clear identification (+)
  - Partial equilibrium (-)
  - Impossible to evaluate large changes (-)
  - Hard to do scenario analysis (-)

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- Recently, **quantitative spatial equilibrium** models (QSE) have become increasingly popular
  - Given the model structure, one may evaluate large changes in spatial structure
  - Model complex spatial interactions

# For what research questions are QSE models useful?



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- Recently, **quantitative spatial equilibrium** models (QSE) have become increasingly popular
  - Given the model structure, one may evaluate large changes in spatial structure
  - Model complex spatial interactions
  
- Useful when interested in:
  - Transport infrastructure investments
  - Sorting/gentrification
  - Evaluating large place-based policies
  - (Changes in) agglomeration economies



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- Learn about a key contribution of Ahlfeldt *et al.* (2015) [ARSW]
  - QSE of Berlin's Urban Spatial Structure
  - Use the Berlin Wall as a quasi experiment to identify the importance of agglomeration economies
  - *Econometric Society Frisch Medal Award*

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- **I have applied similar models in recent papers:**
  - Koster, H.R.A. (2023). The Welfare Effects of Greenbelt Policy. *Economic Journal*, forthcoming
  - Dericks, G., Koster, H.R.A. (2021). The Billion Pound Drop: The Blitz and Agglomeration Economies in London. *Journal of Economic Geography*, 21(6): 869-897
  - Koster, H.R.A, Hayakawa, K., Tabuchi, T., Thisse, J.-F. (2023). High-speed rail and the spatial distribution of economic activity: Evidence from Japan's Shinkansen. RIETI Working Paper.

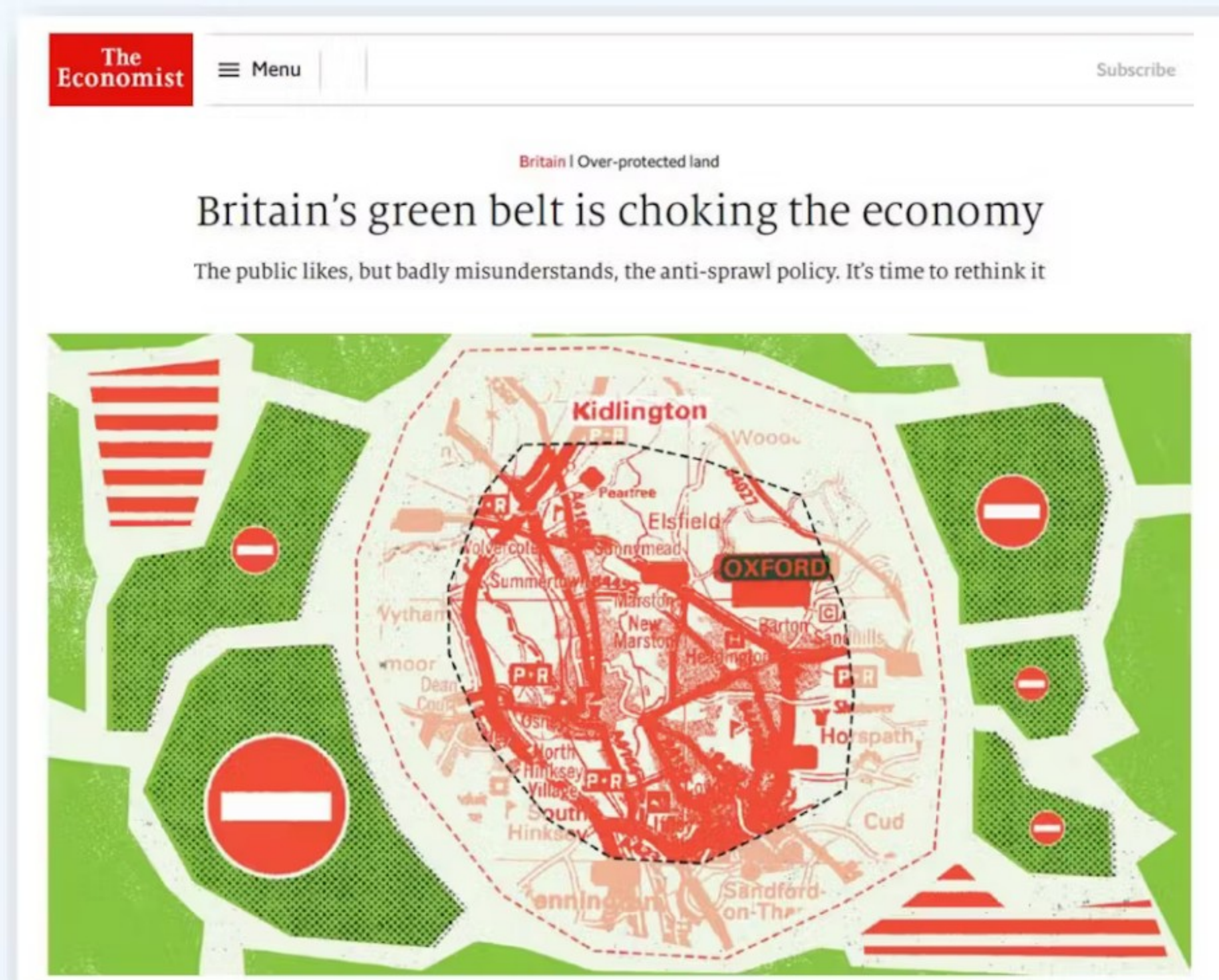
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- The advantages of ARSW
  - Use **commuting flows** to identify key parameters
  - Easy **recursive estimation** using standard regression techniques
  - **Proper identification** of model parameters (rather than calibration...)

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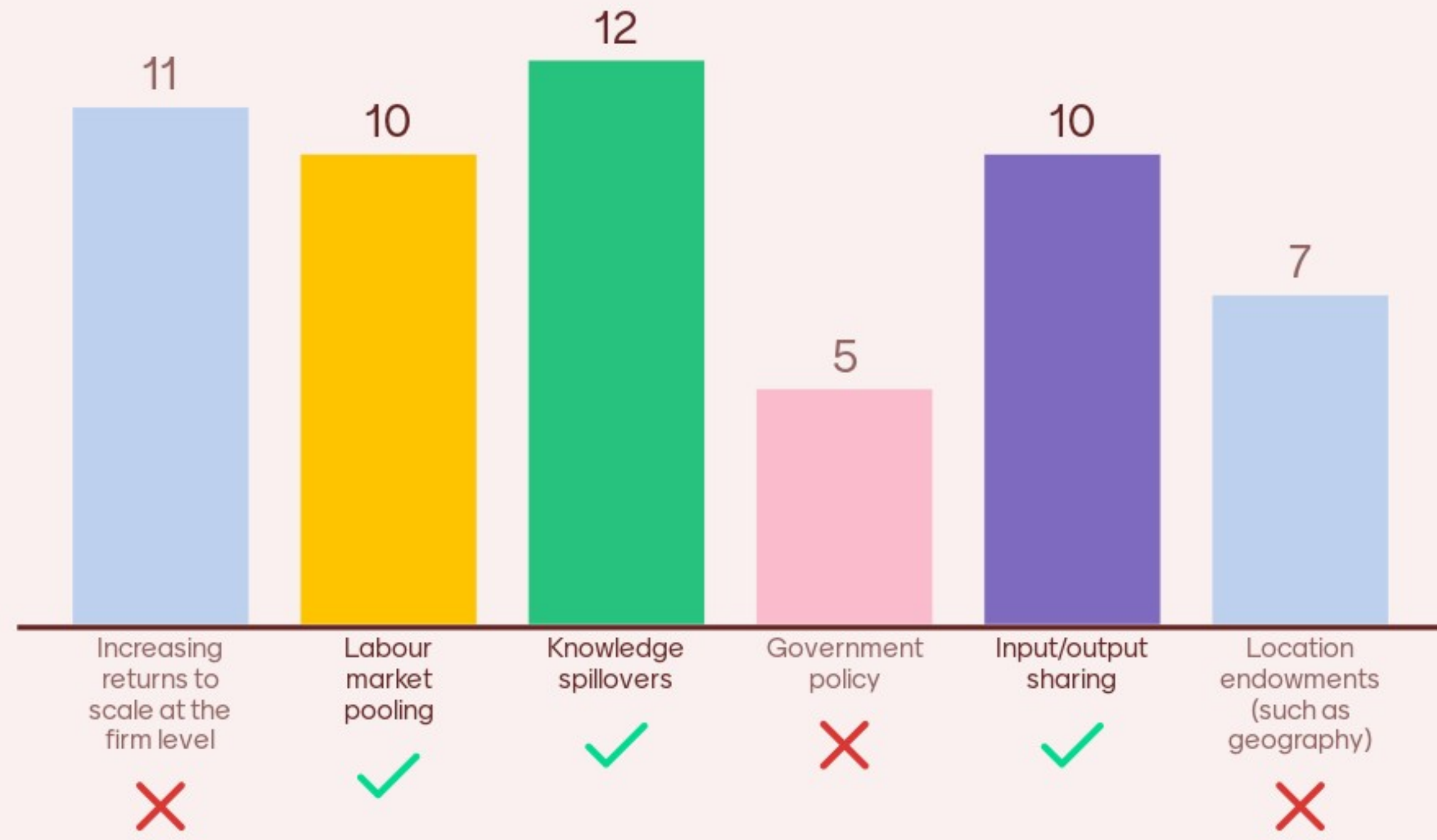
- **Goals of this lecture**

1. **You should understand the model structure of ARSW**
2. **You should be able to estimate the ARSW model**
3. **You should understand the pros and cons of applying the ARSW model**

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- **Main elements:**
  - **CD-Utility of workers** dependent on residential location  $i$  and workplace  $j$
  - **CD-Production**
  - **Land market:** land available is given
  - Production and workers are linked via **commuting**
  - Production benefits from **agglomeration economies**
  - Workers may benefit/lose from **residential externalities**

# What are sources of agglomeration economies? *(multiple answers may be correct)*





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## Workers

$$U_{ijo} = \frac{B_i Z_{ijo}}{d_{ij}} \left( \frac{c_{ijo}}{\beta} \right)^\beta \left( \frac{\ell_{ijo}}{1-\beta} \right)^{1-\beta}$$

$U_{ijo}$  utility for worker  $o$  living in  $i$  and working in  $j$

$B_i$  amenities

$c_{ijo}$  composite good consumption

$\ell_{ijo}$  residential floor space consumption

$Z_{ijo}$  idiosyncratic component where:

$$F(z_{ijo}) = e^{-T_i E_j z_{ijo}^{-\varepsilon}}$$

where  $T_i$  and  $E_j$  denote average utilities

$d_{ij}$  commuting discount factor:  $e^{\kappa\tau_{ij}}$

## Workers

- **Budget constraint**

$$w_j = Q_i \ell_{ijo} + c_{ijo}$$

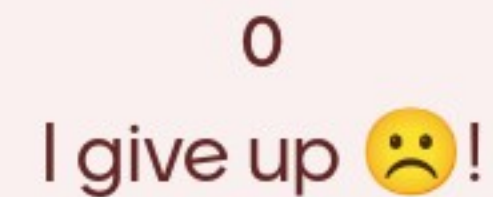
where  $Q_i$  are floor space prices

- **The indirect utility is given by:**

$$u_{ijo} = B_i(w_j e^{-\kappa \tau_{ij}}) Q_i^{\beta-1} z_{ijo}$$

Given  $U_{jio} = \frac{B_i z_{ijo}}{d_{ij}} \left( \frac{c_{ijo}}{\beta} \right)^\beta \left( \frac{l_{ijo}}{1-\beta} \right)^{1-\beta}$ , derive

the indirect utility.



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## Workers

### 1. Set up Lagrange:

$$\mathcal{L} = U_{ij0} + \lambda(w_j - Q_i \ell_{ij0} - c_{ij0})$$

### 2. Derive FOCs:

$$\frac{\partial \mathcal{L}}{\partial c_{ij0}} = \frac{B_i z_{ij0}}{d_{ij}} \left(\frac{c_{ij0}}{\beta}\right)^{\beta-1} \left(\frac{\ell_{ij0}}{1-\beta}\right)^{1-\beta} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \ell_{ij0}} = \frac{B_i z_{ij0}}{d_{ij}} \left(\frac{c_{ij0}}{\beta}\right)^{\beta} \left(\frac{\ell_{ij0}}{1-\beta}\right)^{-\beta} - \lambda Q_i = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w_j - Q_i \ell_{ij0} - c_{ij0} = 0$$

### 3. Using FOC (3) and (2):

$$\frac{\left(\frac{c_{ij0}}{\beta}\right)^{\beta-1} \left(\frac{\ell_{ij0}}{1-\beta}\right)^{1-\beta}}{\left(\frac{c_{ij0}}{\beta}\right)^{\beta} \left(\frac{\ell_{ij0}}{1-\beta}\right)^{-\beta}} = \frac{1}{Q_i}$$

$$\frac{\beta \ell_{ij0}}{(1-\beta)c_{ij0}} = \frac{1}{Q_i}$$

$$\ell_{ij0} = \frac{(1-\beta)c_{ij0}}{\beta Q_i}$$

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## Workers

4. Plug  $\ell_{ijo} = \frac{(1-\beta)c_{ijo}}{\beta Q_i}$  in the budget constraint:

$$w_j - Q_i \frac{(1-\beta)c_{ijo}}{\beta Q_i} - c_{ijo} = 0$$

$$w_j - \left(\frac{1-\beta}{\beta} + 1\right) c_{ijo} = 0$$

$$c_{ijo}^* = \beta w_j$$

5. Plug  $c_{ijo}^*$  in  $\ell_{ijo} = \frac{(1-\beta)c_{ijo}}{\beta Q_i}$  to obtain:

$$\ell_{ijo}^* = \frac{(1-\beta)w_j}{Q_i}$$

6. Plug  $c_{ijo}^*$  and  $\ell_{ijo}^*$  in the utility function:

$$u_{ijo} = \frac{B_i z_{ijo}}{d_{ij}} w_j Q_i^{\beta-1} = \bar{u}_{ijo} z_{ijo}$$

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## Workers

- **Fréchet shock on commuting**
  - Eaton and Kortum (2002) in trade
  - Captures idiosyncratic preferences for living in  $i$  and working in  $j$

- **Hence:**

$$\begin{aligned}\pi_{ij} &= \frac{T_i E_j \bar{u}_{ij}^\varepsilon}{\sum_{r=1}^S \sum_{s=1}^S T_r E_s \bar{u}_{rs}^\varepsilon} \\ &= \frac{T_i E_j \left( B_i (w_j e^{-\kappa \tau_{ij}}) Q_i^{\beta-1} \right)^\varepsilon}{\sum_{r=1}^S \sum_{s=1}^S T_r E_s \left( B_r (w_s e^{-\kappa \tau_{rs}}) Q_r^{\beta-1} \right)^\varepsilon}\end{aligned}$$

- **Conditional on living in  $i$ , the commuting probability to  $j$  is given by:**

$$\pi_{ij|i} = \frac{E_j (w_j e^{-\kappa \tau_{ij}})^\varepsilon}{\sum_{s=1}^S E_s (w_s e^{-\kappa \tau_{is}})^\varepsilon}$$

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## Workers

- **Commuting market clearing condition:**

$$H_{Mj} = \sum_{i=1}^S \pi_{ij|i} H_{Ri}$$

$$H_{Mj} = \sum_{i=1}^S \frac{E_j(w_j e^{-\kappa\tau_{ij}})^\varepsilon}{\sum_{s=1}^S E_s(w_s e^{-\kappa\tau_{is}})^\varepsilon} H_{Ri}$$

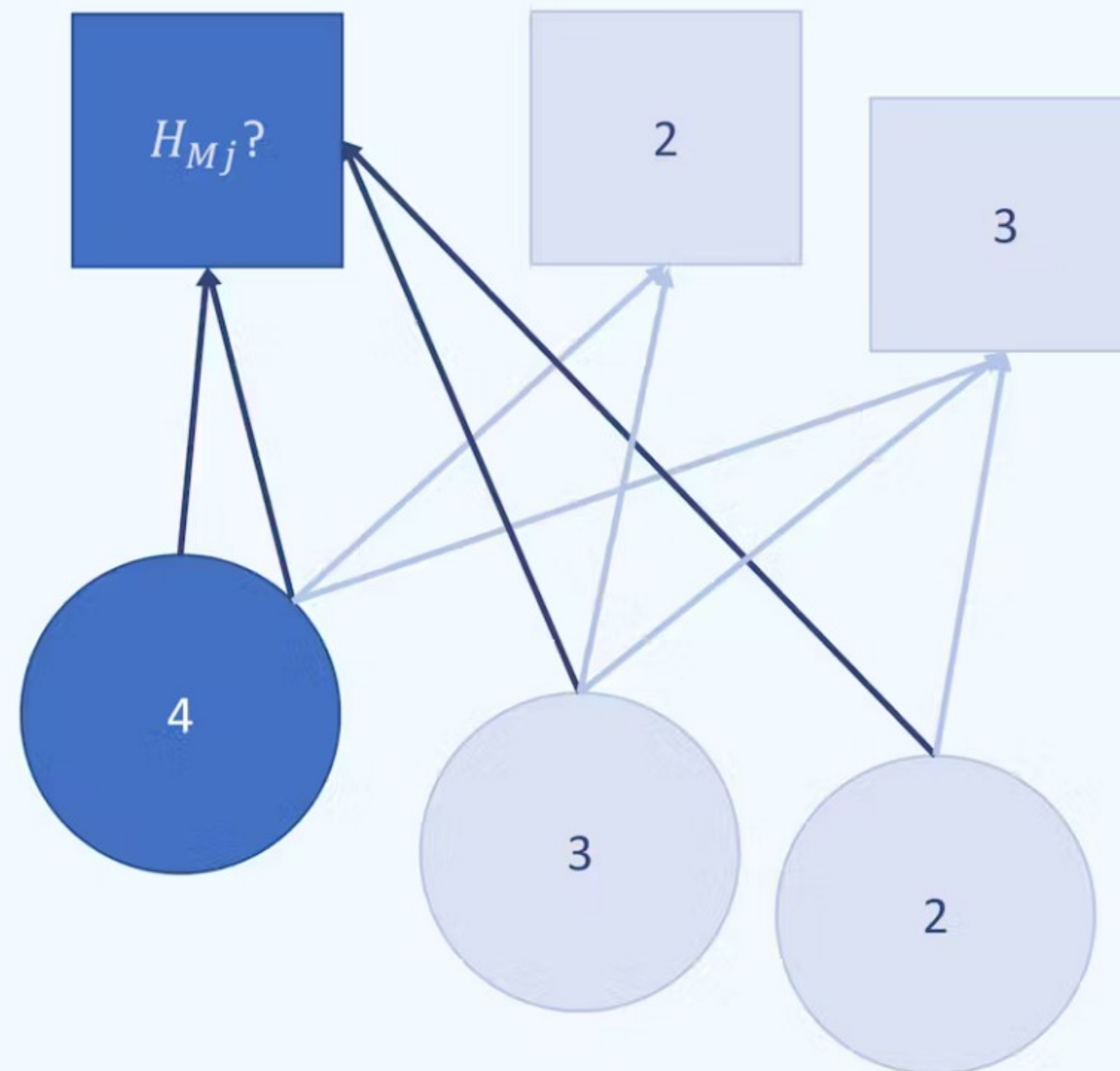
$H_{Mj}$  workers at  $j$

$H_{Ri}$  residents at  $i$

## Workers

- **Commuting market clearing condition:**

$$H_{Mj} = \sum_{i=1}^S \pi_{ij|i} H_{Ri}$$





Given  $H_{Mj} = \sum_{i=1}^S \pi_{ij|i} H_{Ri}$  What happens if  $w_j$

increases?



✓ Working location  $j$  becomes more attractive so that the probability on in-commuters from  $i$  rises for all  $i$



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## Workers

- **Spatial equilibrium** so that workers have the same *expected utility everywhere*:

$$\mathbb{E}[\bar{u}] = \Gamma\left(\frac{\varepsilon - 1}{\varepsilon}\right) \left[ \sum_{r=1}^S \sum_{s=1}^S T_r E_s \left( B_r (w_s e^{-\kappa \tau_{rs}}) Q_r^{\beta-1} \right)^\varepsilon \right]^{\frac{1}{\varepsilon}}$$

where  $[\cdot]$  is the denominator of  $\pi_{ij}$  and  $\Gamma(\cdot)$  is the gamma function

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- **Production**

$$y_j = A_j H_{Mj}^\alpha L_{Mj}^{1-\alpha}$$

$y_j$       **output**

$A_j$       **final goods productivity**

$H_{Mj}$     **labour input**

$L_{Mj}$     **commercial floor space used**

**Then:**

$$q_j = (1 - \alpha) \left( \frac{\alpha}{w_j} \right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}$$

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- **Floor space market clearing**

- $Q_i = \max(q_i, Q_i)$
- $\theta_i = 1$       **if**  $q_i > Q_i$
- $\theta_i \in [0,1]$     **if**  $q_i = Q_i$
- $\theta_i = 0$       **if**  $q_i < Q_i$

- **Floor space market clearing for households**

$$\mathbb{E}[\ell_i]H_{Ri} = \frac{(1 - \beta)\mathbb{E}[w_s|i]}{Q_i}H_{Ri} = (1 - \theta_i)L_i$$

- **Floor space clearing for firms**

$$\left(\frac{(1 - \alpha)A_j}{q_j}\right)^{\frac{1}{\alpha}}H_{Mj} = \theta_jL_j$$

- **Total floor space demand equals total supply:**

$$(1 - \theta_i)L_{Ri} + \theta_iL_{Mi} = L_i = K_i^\mu M_i^{1-\mu}$$

where  $K_i$  is the land available at  $i$  and  $M_i^{1-\mu}$  is the density of development

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- **General equilibrium with fixed  $B_i$  and  $A_i$**
- **Equilibrium is determined by**
  - Population mobility (*i.e.* expected utility)
  - Residential choice probability
  - Workplace choice probability
  - Commercial land market clearing
  - Residential land market clearing
  - Profit maximization
  - Zero profit
  - No-arbitrage between alternative land uses
- **ARSW prove **existence** and **uniqueness** of equilibrium**

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- **Allow for endogenous agglomeration forces**

- **Production externalities:**

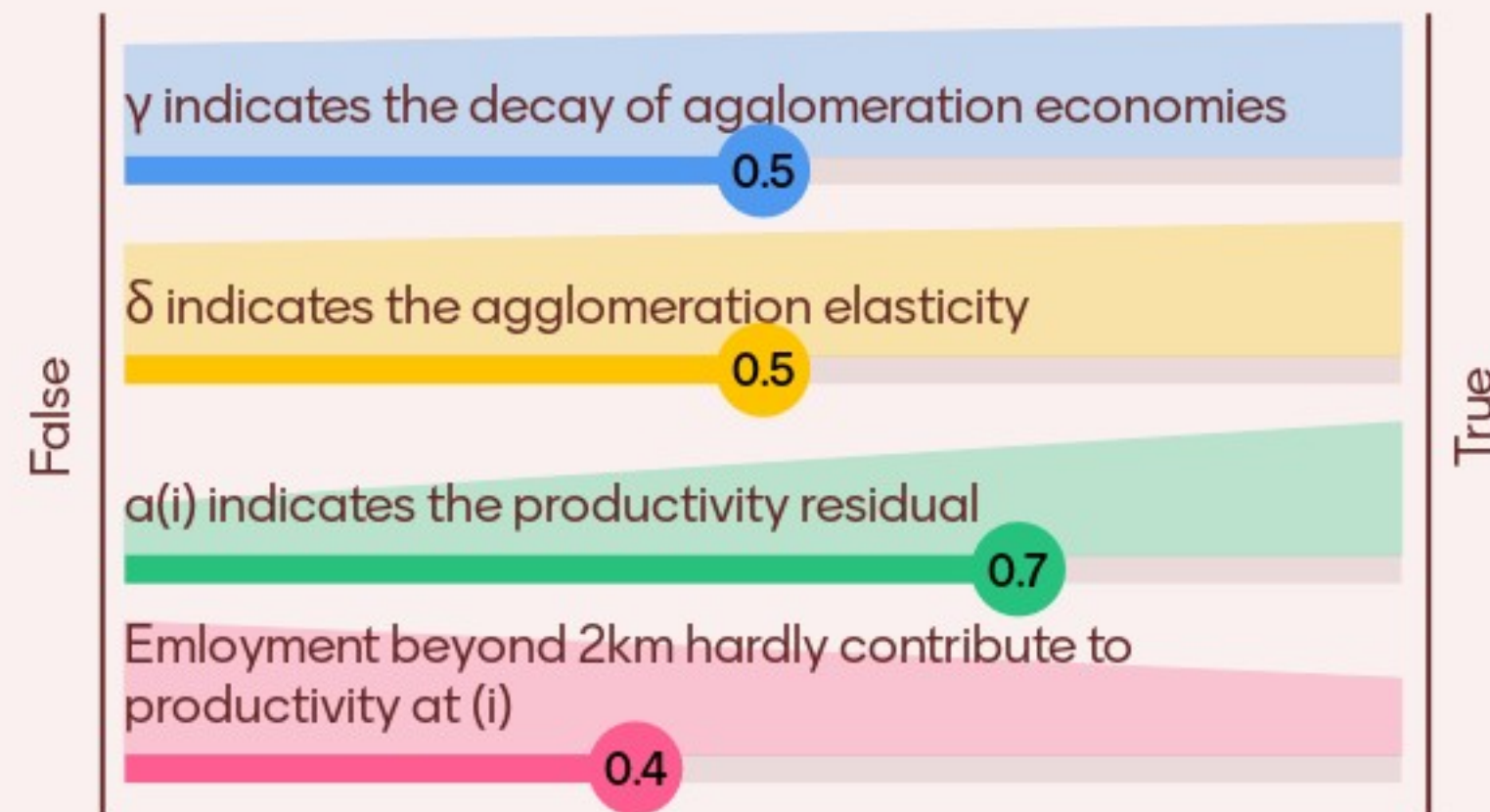
$$A_i = a_i \left( \sum_{s=1}^S e^{-\delta\tau_{is}} \frac{H_{Ms}}{K_s} \right)^\lambda$$

- **Residential externalities:**

$$B_i = b_i \left( \sum_{s=1}^S e^{-\rho\tau_{is}} \frac{H_{Rs}}{K_s} \right)^\eta$$

# Consider productivity levels

$A_i = a_i \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right)^\gamma$ . What statements are true?





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- **Allow for endogenous agglomeration forces**

- **Production externalities:**

$$A_i = a_i \left( \sum_{s=1}^S e^{-\delta\tau_{is}} \frac{H_{Ms}}{K_s} \right)^\lambda$$

- **Residential externalities:**

$$B_i = b_i \left( \sum_{s=1}^S e^{-\rho\tau_{is}} \frac{H_{Rs}}{K_s} \right)^\eta$$

- **Please explain intuition behind  $A_i$  and  $B_i$**

*Density leads to higher  $A_i$  and  $B_i$  if  $\{\lambda, \eta\} > 0$ , but with spatial decay indicated by  $\delta$  and  $\rho$*

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- **Recovering  $A_i$  and  $B_i$  from the model**
  - **There are closed forms of  $A_i$  and  $B_i$ , *up to multiplication constants***
  - **‘Structural residuals’**

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- **Recovering  $A_i$  and  $B_i$  from the model**

- **Productivity (using zero profit condition)**

$$A_i = (1 - \alpha)^{1-\alpha} \alpha^{-\alpha} Q_j^{1-\alpha} W_j^\alpha$$

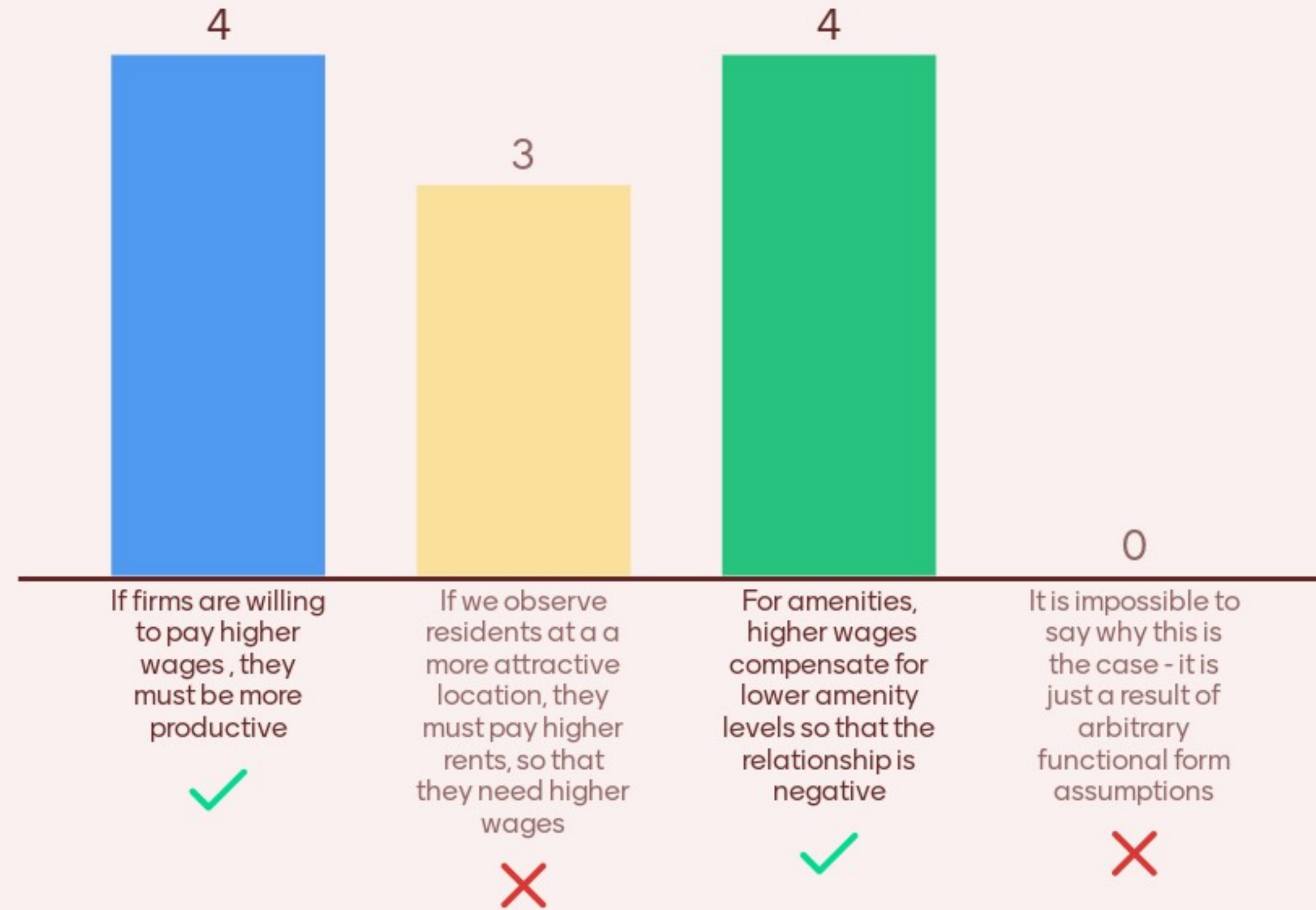
- **Amenities (using the expected utility):**

$$\frac{\tilde{B}_i}{\bar{\tilde{B}}} = \left( \frac{H_{Ri}}{\bar{H}_R} \right)^{\frac{1}{\varepsilon}} \left( \frac{Q_i}{\bar{Q}} \right)^{1-\beta} \left( \frac{W_i}{\bar{W}} \right)^{-\frac{1}{\varepsilon}}$$

where bars denote geometric means and  $W_i$  is the expected wage:

$$W_i = \sum_{s=1}^S E_s (w_j e^{-\kappa \tau_{ij}})^\varepsilon$$

Why is the relationship between wages and productivities,  $A_i$ , positive, while it is negative for amenities,  $B_i$ ? (multiple answers may be correct)



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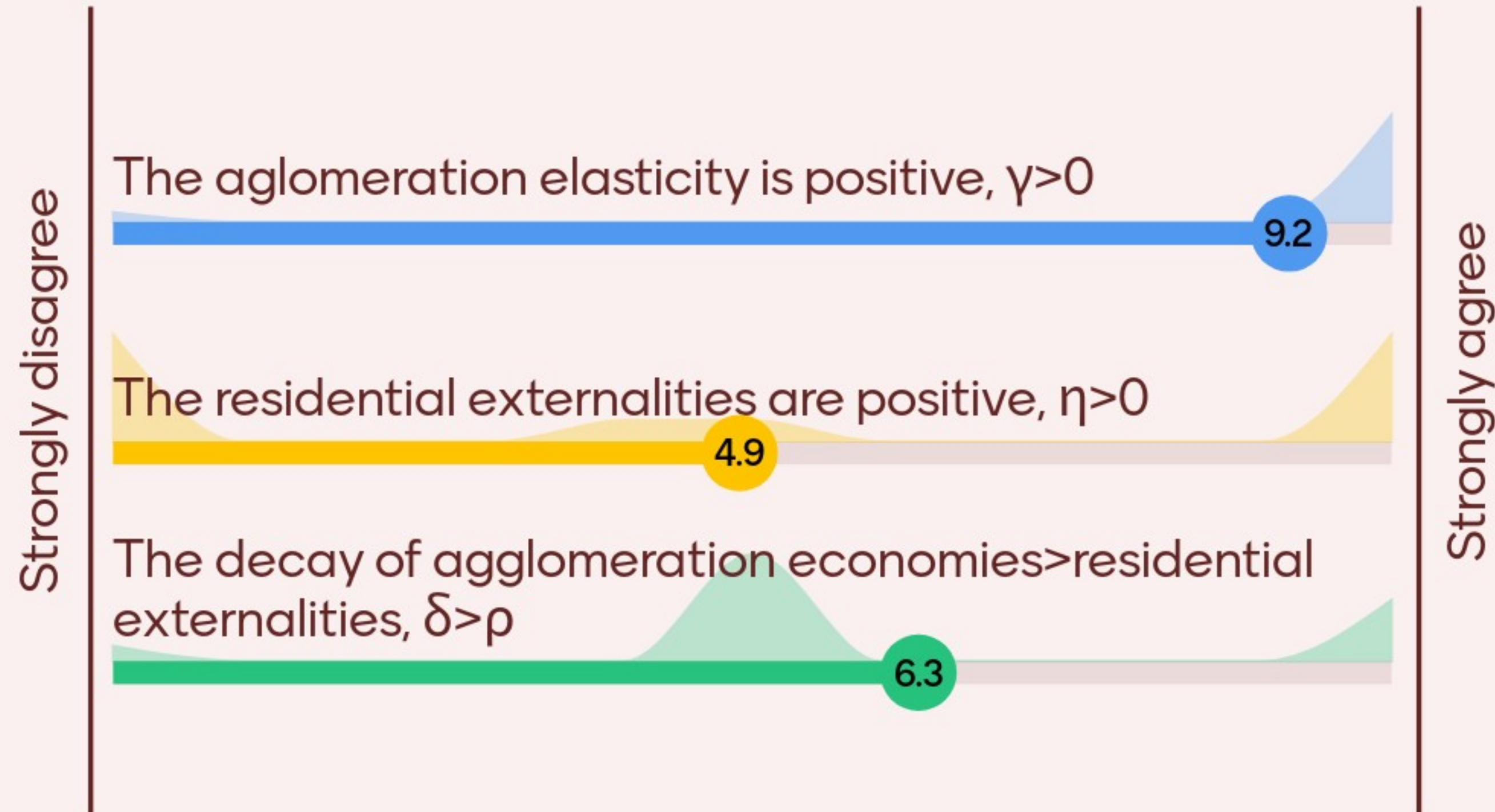
❓ **Does this make sense?**

- If wages/rents  $\uparrow$ , productivity must be higher
- If wages  $\uparrow$ , amenities  $\downarrow$ , if rents or  $H_{Ri}$   $\uparrow$ , amenities higher (see Roback, 1982)

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- **We aim to estimate**
  - $\kappa\varepsilon > 0$  **commuting time elasticity**
  - $\varepsilon > 1$  **utility dispersion parameter**
  
  - $\lambda$  **productivity elasticity**
  - $\delta > 0$  **productivity decay**
  
  - $\eta$  **residential elasticity**
  - $\rho > 0$  **residential decay**

# What are your hypotheses with respect to the parameters?



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- **We aim to estimate**

$\kappa\varepsilon > 0$  **commuting time elasticity**

$\varepsilon > 1$  **utility dispersion parameter**

$\lambda$  **productivity elasticity**

$\delta > 0$  **productivity decay**

$\eta$  **residential elasticity**

$\rho > 0$  **residential decay**

❓ **What are the expected signs of  $\lambda$  and  $\eta$ ?**

*Positive for  $\lambda$  (agglomeration economies),  $\eta$  is unsure*

❓ **Would you expect  $\delta > \rho$  or  $\delta < \rho$ ?**

*I would expect that  $\delta < \rho$ , the decay of neighbourhood interactions to be more local*



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- **Estimate the parameters using its recursive structure**
  1. Estimate commuting gravity equation
  2. Back out wages
  3. Obtain utility dispersion parameter  $\varepsilon$
  4. Obtain amenities and productivity
  5. Regress productivity on worker density
  6. Regress amenities on residential density

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- **Important:**
  - **Everything is identified up to multiplication constants**
  - **We cannot say much about *absolute* utility levels**
  
- **Good identification strategies are key to obtain correct model parameters!**
  - *Like in reduced-form estimation*
  - **Model can be seen as a collection of reduced-form estimations**
  - **... much better than in structural models in the past**

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## 1. Estimate gravity equation:

We can write  $\pi_{ij}$  as:

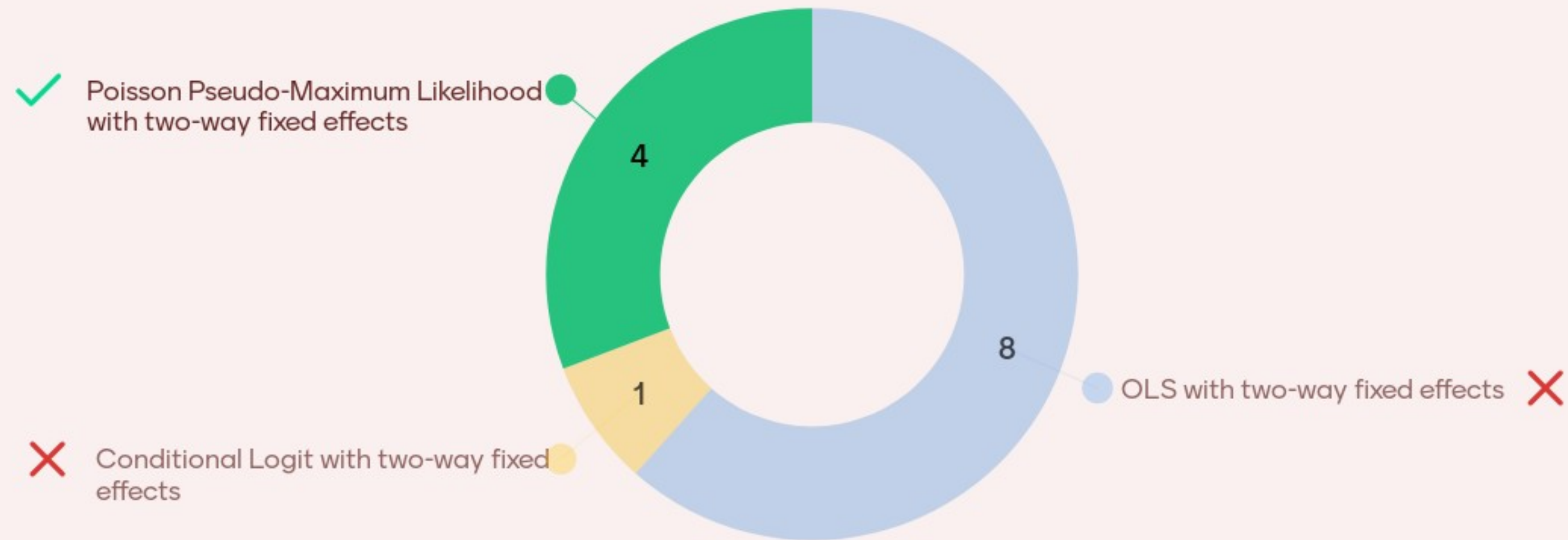
$$\log \pi_{ij} H = -\kappa \tau_{ij} + v_i + v_j$$

which identifies  $\kappa = \varepsilon \kappa$

- This is a standard gravity equation, as  $\pi_{ij} H$  represents the commuting flow between  $i$  and  $j$ 
  - $v_i$  and  $v_j$  absorb  $T_i, B_i, Q_i, E_j$ .

# How would you estimate

$$\pi_{ij}H = -\kappa\tau_{ij} + v_i + v_j?$$



## 1. Estimate gravity equation:

We can write  $\pi_{ij}$  as:

$$\log \pi_{ij} H = -\kappa \tau_{ij} + v_i + v_j$$

which identifies  $\kappa = \varepsilon \kappa$

- This is a standard gravity equation, as  $\pi_{ij} H$  represents the commuting flow between  $i$  and  $j$ 
  - $v_i$  and  $v_j$  absorb  $T_i, B_i, Q_i, E_j$ .

### Q How would you estimate this?

*Dependent variable is count variable – Poisson regression with two-way fixed effects; OR OLS with two-way fixed effects excluding zero flows (which is not very attractive in my opinion)*

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2. Given data on  $H_{Mj}$ ,  $H_{Rj}$  and  $\hat{\kappa}$ , we can obtain transformed wages  $\omega_j = (E_j w_j)^\varepsilon$  for each location:

$$H_{Mj} = \sum_{i=1}^S \frac{\omega_j e^{-\hat{\kappa}\tau_{ij}}}{\sum_{s=1}^S \omega_s e^{-\hat{\kappa}\tau_{is}}} H_{Ri}$$

- There exists a unique vector of  $\omega_j$  that ensures that the commuting market clearing condition holds
  
- Use Newton-Raphson procedure
  - $\omega_{j,0} = 1$
  - $\omega_{j,r+1} = \omega_{j,r} \frac{H_{Mj}}{\hat{H}_{Mj,r}}$

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3. Given aggregate data on wages  $w_i$ ,  $\hat{\omega}_j$  and  $\hat{\kappa}$ , we can back out  $\varepsilon$

$$\text{var}(\log \omega_j) = \text{var}\left(\log(E_j w_j)^\varepsilon\right) = \varepsilon \text{var}(\log(w_j))$$

Hence,

$$\mathbb{E} \left[ \text{var}(\log(w_i)) - \frac{1}{\varepsilon} \text{var}(\log \omega_j) \right] = 0$$

→ Use OLS (without a constant) to obtain  $\varepsilon$

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4. Given data on  $H_{Mi}$ ,  $H_{Ri}$ ,  $\hat{\nu}$ ,  $\hat{\varepsilon}$ ,  $\hat{\omega}_j$  and data on floor space prices  $Q_j$  we recover  $A_i$  and  $B_i$  (up to a constant)

- Productivities in logs (eq. 27):

$$\log \tilde{A}_i = \log \tilde{a}_i + (1 - \alpha) \log Q_i + \frac{\alpha}{\hat{\varepsilon}} \log \omega_i$$

- Amenities in logs (eq. 28):

$$\log \tilde{B}_i = \log \tilde{b}_i + \frac{1}{\hat{\varepsilon}} \log H_{Ri} + (1 - \beta) \log Q_i - \frac{1}{\hat{\varepsilon}} \log W_i$$



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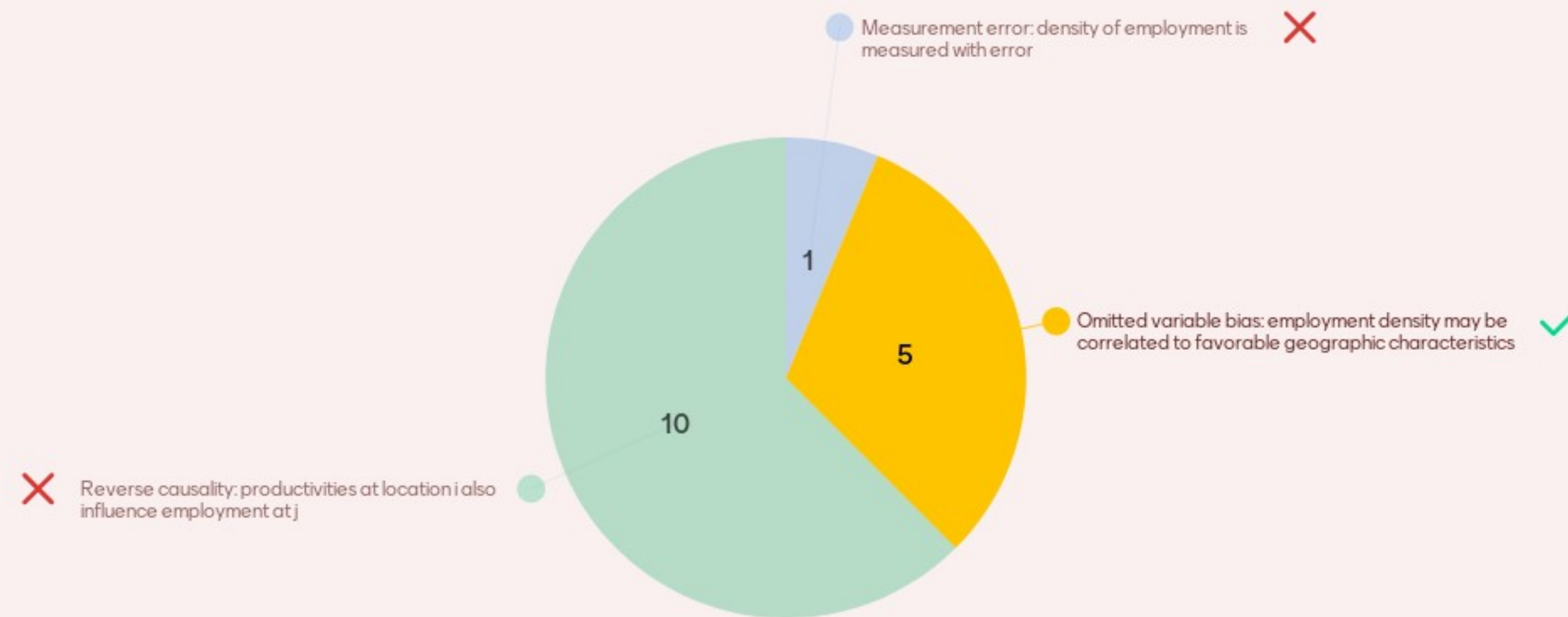
## 5. Production externalities

- We have data on  $H_{Ms}$  and have data on  $K_s$ :

$$\log \hat{A}_i = \log \tilde{a}_i + \lambda \log \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right) + \xi_i$$

# What is the main endogeneity issue when estimating

$$\log \hat{A}_i = \tilde{a}_i + \gamma \log \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right) + \xi_i?$$



## 5. Production externalities

- We have data on  $H_{Ms}$  and have data on  $K_s$ :

$$\log \hat{A}_i = \log \tilde{a}_i + \lambda \log \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right) + \xi_i$$

❓ **What are endogeneity issues here and how would you address them?**

- *Density is endogenous and correlated to unobserved locational endowments*
- *IV – use changes in density due to Berlin Wall*
- *Use historic instruments*

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## 5. Production externalities

- **We have data on  $H_{Ms}$  and have data on  $K_s$ :**

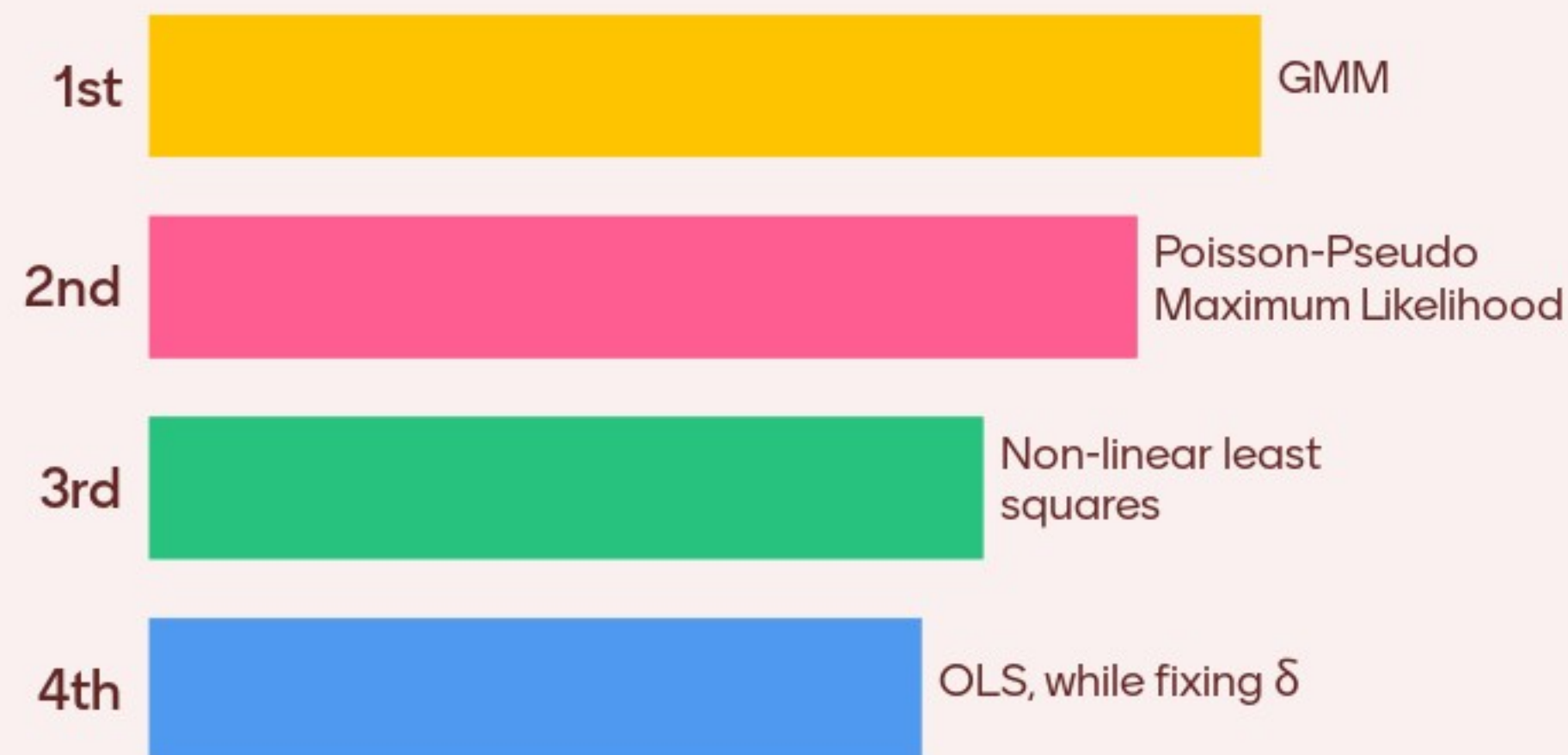
$$\log \hat{A}_i = \log \tilde{a}_i + \lambda \log \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right) + \xi_i$$

- **Use changes in density due to Berlin Wall**

$$\Delta \log \hat{A}_{it} = \Delta \log \tilde{a}_{it} + \lambda \Delta \log \left( \sum_{s=1}^S e^{-\delta \tau_{is,t}} \frac{H_{Ms,t}}{K_{s,t}} \right) + \Delta \xi_{it}$$

How would you estimate  $\Delta \log \hat{A}_{i,t} = \Delta \tilde{a}_{i,t} +$

$$\gamma \Delta \log \left( \sum_{s=1}^S e^{-\delta \tau_{is,t}} \frac{H_{Ms,t}}{K_s} \right) + \Delta \xi_{i,t} ? \text{ Please rank:}$$



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## 5. Production externalities

- We have data on  $H_{Ms}$  and have data on  $K_s$ :

$$\log \hat{A}_i = \log \tilde{a}_i + \lambda \log \left( \sum_{s=1}^S e^{-\delta \tau_{is}} \frac{H_{Ms}}{K_s} \right) + \xi_i$$

- Use changes in density due to Berlin Wall

$$\Delta \log \hat{A}_{it} = \Delta \log \tilde{a}_{it} + \lambda \Delta \log \left( \sum_{s=1}^S e^{-\delta \tau_{is,t}} \frac{H_{Ms,t}}{K_{s,t}} \right) + \Delta \xi_{it}$$

### Q How would you estimate this?

- Standard OLS does not work because of decay parameter... maybe fix  $\delta$ ?
- Use GMM or Nonlinear Least Squares

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## 6. Residential externalities

- Similarly, we have data on  $H_{RS}$  and estimated  $K_S$ :

$$\log \hat{B}_i = \log \tilde{b}_i + \eta \log \left( \sum_{s=1}^S e^{-\rho\tau_{is}} \frac{H_{RS}}{K_S} \right) + \xi_i$$

- Use again variation in density due to Berlin Wall...

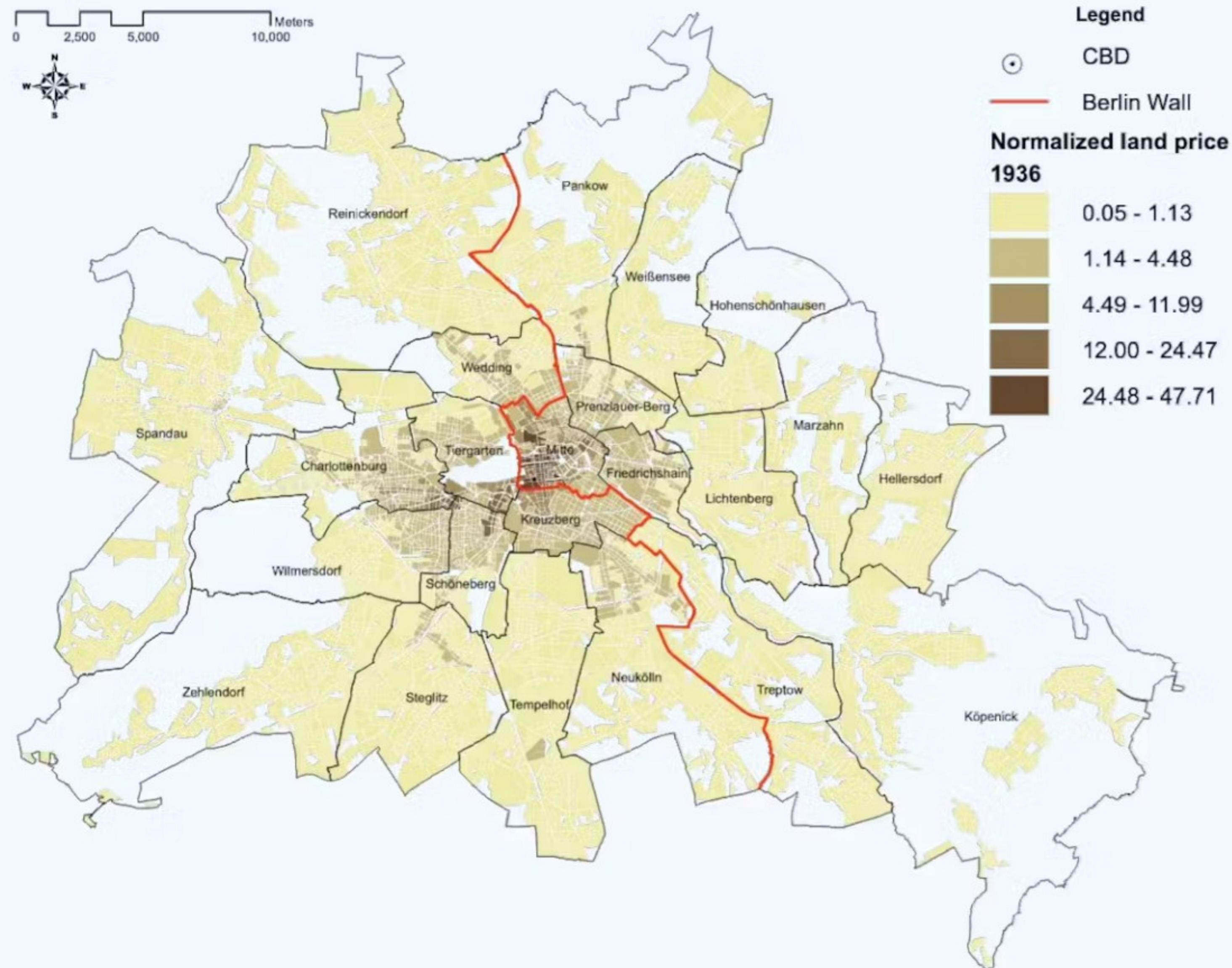
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- Ahlfeldt *et al.* use a GMM approach to estimate all the parameters in one go
- In principle this should deliver (more or less) the same estimates
- Only possible with linear (no PPML) gravity model without too many observations



# 4. Empirical evidence

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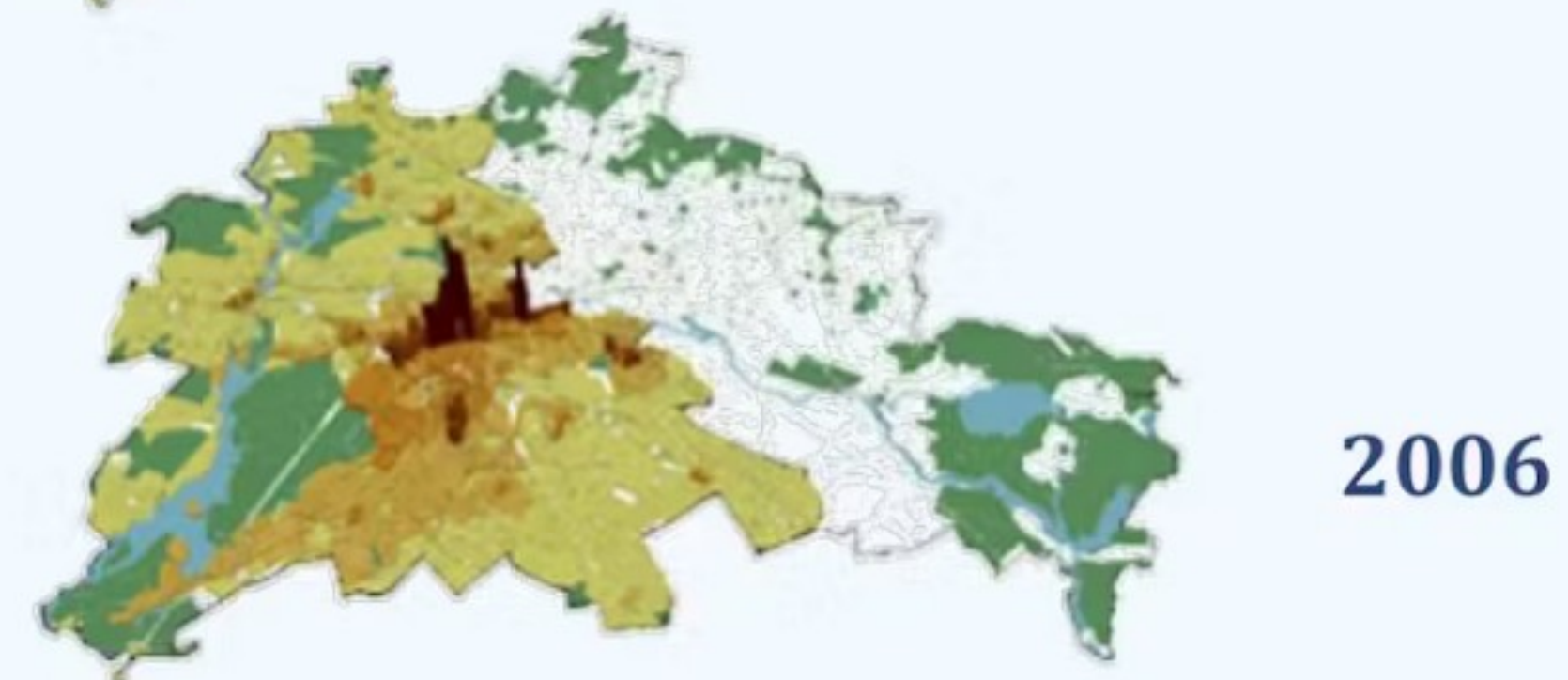
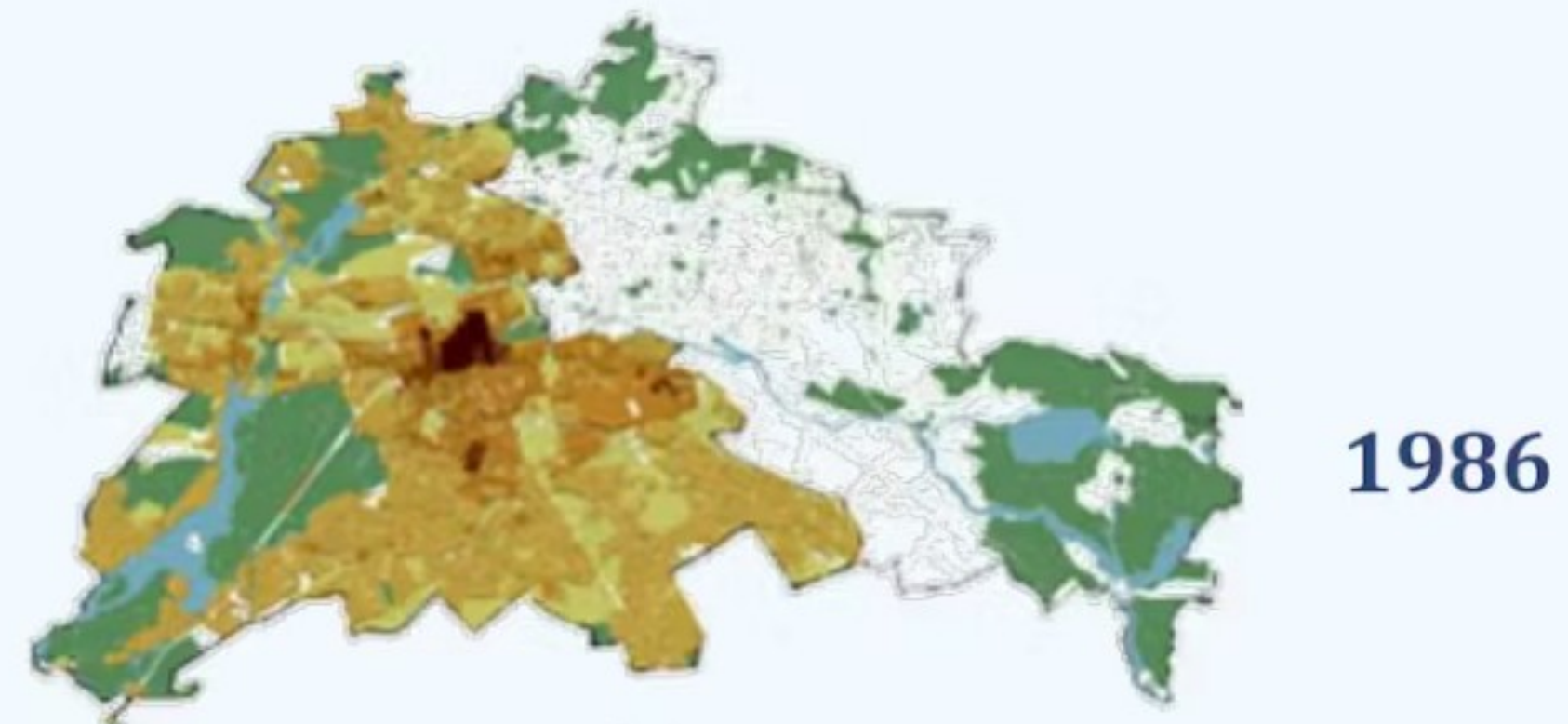
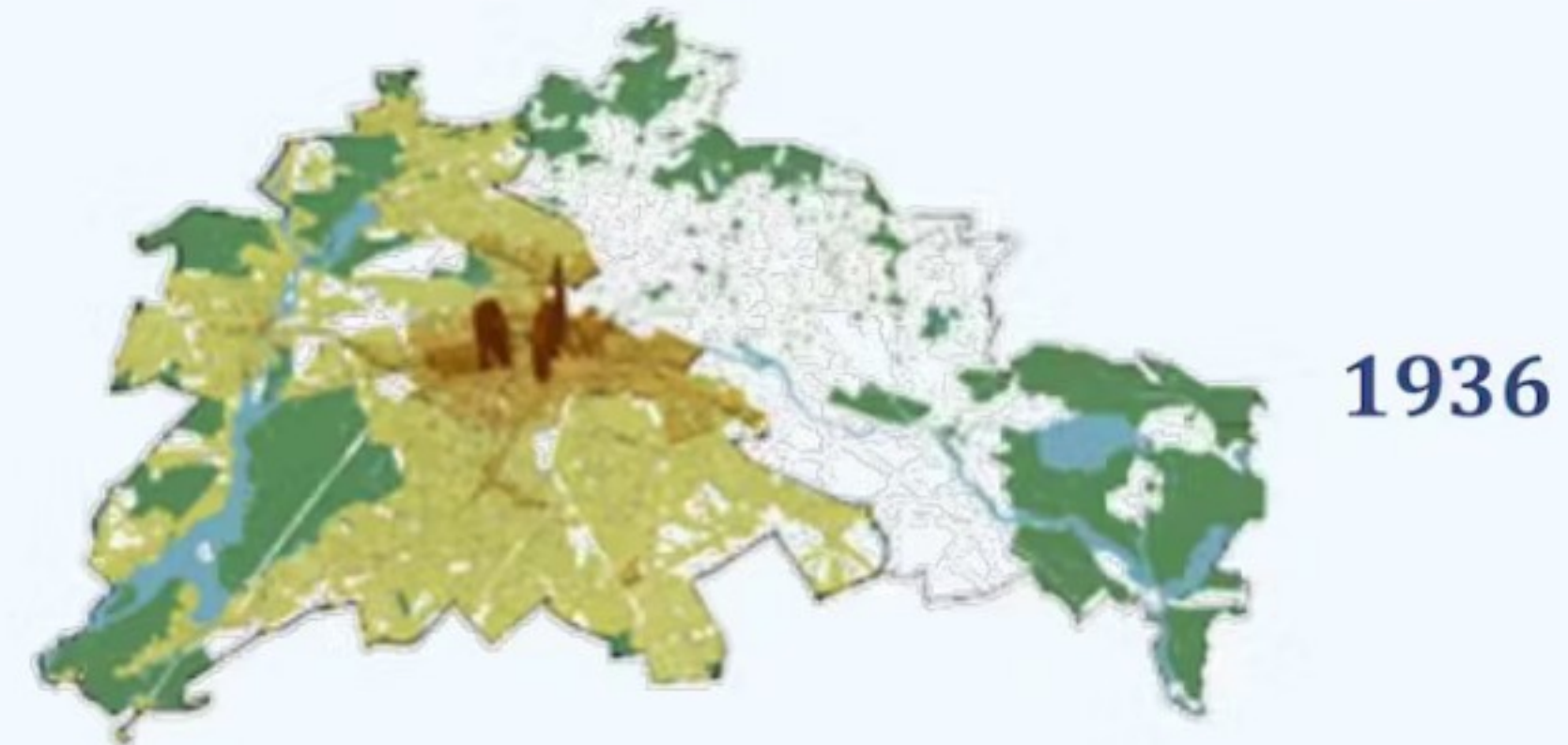


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- **They use block data from 1936, 1986 and 2006**
  - **15,937 blocks, 9,000 in West-Berlin**
  - **Land values, commuting times, block characteristics**

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## ▪ Graphical illustration of changes



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- **Let's first consider the reduced-form evidence**
  - **This is an important starting point of any analysis!**

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### ▪ Three regressions for division and reunification

- $\Delta \log Q_i = \alpha + \sum_{k=1}^K \beta_k I_{ik} + \gamma \log M_i + \epsilon_i$
- $\Delta \log EmpR_i = \check{\alpha} + \sum_{k=1}^K \check{\beta}_k I_{ik} + \check{\gamma} \log M_i + \check{\epsilon}_i$
- $\Delta \log EmpW_i = \tilde{\alpha} + \sum_{k=1}^K \tilde{\beta}_k I_{ik} + \tilde{\gamma} \log M_i + \tilde{\epsilon}_i$

$I_{ik}$       **Within a distance 500m bands of the pre-war CBD**

$M_i$       **time-invariant block characteristics**

$Q_i$       **land values**

$EmpR_i$     **~ Household density**

$EmpW_i$     **Workplace density**

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## Results from division

Table – RENTS, EMPLOYMENT AND THE BERLIN WALL

	Division		Reunification	
	Rents ( <i>log</i> )	Empl. ( <i>log</i> )	Rents ( <i>log</i> )	Empl. ( <i>log</i> )
	(1)	(2)	(3)	(4)
CBD 0-500m	-0.567*** (0.071)	-0.691* (0.408)	0.408*** (0.090)	1.574*** (0.479)
CBD 500-1000m	-0.422*** (0.047)	-1.253*** (0.293)	0.289*** (0.096)	0.684** (0.326)
CBD 1000-1500m	-0.306*** (0.039)	-0.341 (0.241)	0.120*** (0.033)	0.326 (0.216)
CBD 1500-2000m	-0.207*** (0.033)	-0.512*** (0.199)	-0.031 (0.023)	0.336** (0.161)
CBD 2000-2500m	-0.139*** (0.024)	-0.436*** (0.151)	0.018 (0.015)	0.114 (0.118)
CBD 2500-3000m	-0.125*** (0.019)	-0.280*** (0.130)	-0.000 (0.012)	0.049 (0.095)
District fixed effects	Yes	Yes	Yes	Yes
Number of observations	6,260	2,844	7,050	5,602
Kleibergen-Paap <i>F</i> -statistic	0.51	0.12	0.32	0.03

Notes: Data on pre-division is from 1936, during the division it is from 1986 and from reunification it is from 2006. Standard errors adjusted for spatial correlation are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.10$ .

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**Q Please interpret the results!**

# What is true with respect to the results? (multiple answers may be correct)





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## Gravity model

TABLE III  
COMMUTING GRAVITY EQUATION<sup>a</sup>

	(1)	(2)	(3)	(4)
	In Bilateral Commuting Probability 2008	In Bilateral Commuting Probability 2008	In Bilateral Commuting Probability 2008	In Bilateral Commuting Probability 2008
Travel Time ( $-\kappa\varepsilon$ )	-0.0697*** (0.0056)	-0.0702*** (0.0034)	-0.0771*** (0.0025)	-0.0706*** (0.0026)
Estimation	OLS	OLS	Poisson PML	Gamma PML
More than 10 Commuters		Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Observations	144	122	122	122
$R^2$	0.8261	0.9059	-	-

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## Structural parameters

TABLE V  
GENERALIZED METHOD OF MOMENTS (GMM) ESTIMATION RESULTS<sup>a</sup>

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ( $\kappa\varepsilon$ )	0.0951*** (0.0016)	0.1011*** (0.0016)	0.0987*** (0.0016)
Commuting Heterogeneity ( $\varepsilon$ )	6.6190*** (0.0939)	6.7620*** (0.1005)	6.6941*** (0.0934)
Productivity Elasticity ( $\lambda$ )	0.0793*** (0.0064)	0.0496*** (0.0079)	0.0710*** (0.0054)
Productivity Decay ( $\delta$ )	0.3585*** (0.1030)	0.9246*** (0.3525)	0.3617*** (0.0782)
Residential Elasticity ( $\eta$ )	0.1548*** (0.0092)	0.0757** (0.0313)	0.1553*** (0.0083)
Residential Decay ( $\rho$ )	0.9094*** (0.2968)	0.5531 (0.3979)	0.7595*** (0.1741)

# What is true with respect to the results? *(multiple answers may be correct)*



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## Implied decay of commuting and externalities

TABLE VI  
EXTERNALITIES AND COMMUTING COSTS<sup>a</sup>

	(1) Production Externalities ( $1 \times e^{-\delta\tau}$ )	(2) Residential Externalities ( $1 \times e^{-\rho\tau}$ )	(3) Utility After Commuting ( $1 \times e^{-\kappa\tau}$ )
0 minutes	1.000	1.000	1.000
1 minute	0.696	0.468	0.985
2 minutes	0.485	0.219	0.971
3 minutes	0.338	0.102	0.957
5 minutes	0.164	0.022	0.929
7 minutes	0.079	0.005	0.902
10 minutes	0.027	0.001	0.863
15 minutes	0.004	0.000	0.802
20 minutes	0.001	0.000	0.745
30 minutes	0.000	0.000	0.642

<sup>a</sup>Proportional reduction in production and residential externalities with travel time and proportional reduction in utility from commuting with travel time. Travel time is measured in minutes. Results are based on the pooled efficient GMM parameter estimates:  $\delta = 0.3617$ ,  $\rho = 0.7595$ ,  $\kappa = 0.0148$ .

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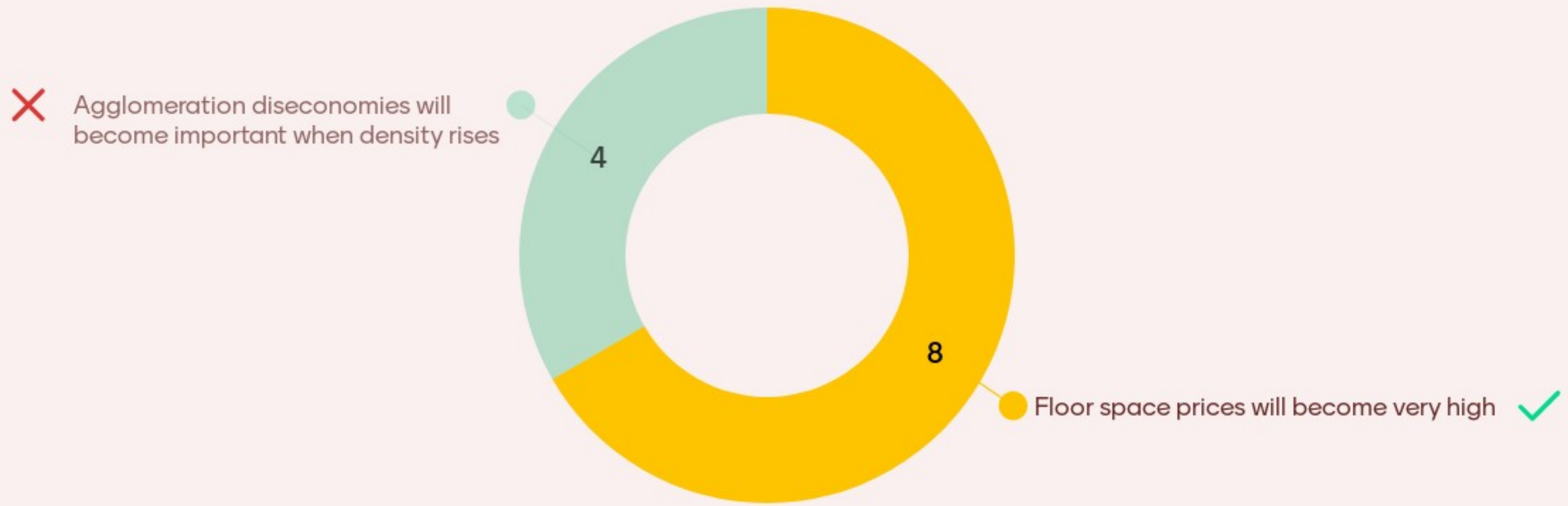
- **Given chosen parameters  $\{\alpha, \beta, \mu\}$  and estimated parameters  $\{\hat{\kappa}, \hat{\varepsilon}, \hat{\lambda}, \hat{\delta}, \hat{\eta}, \hat{\rho}\}$  we can investigate what happens if you change fundamentals**
  
- **The procedure is described in Supplement, pp. 56-57**
  
- **The idea is that you have a change in say travel times  $\tau_{ij}$** 
  - **Due to reunification or division**
  - **Update values iteratively**
$$\{\pi_{ij}, \pi_{ij|i}, H_{Ri}, H_{Mi}, Y_i, \tilde{w}_i, \mathbb{E}[\tilde{w}_s|i], Q_i, \theta_i\}$$

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❓ **Why does the economy not collapse into a point?**

# Why does the economy not collapse into one point?



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1. Use 1936 values to simulate land prices in 1986
  - Simulate **division**
2. Use 1986 values to simulate land prices in 2006
  - Simulate **reunification**

TABLE VII  
COUNTERFACTUALS<sup>a</sup>

	(1) $\Delta \ln QC$ 1936–1986	(5) $\Delta \ln QC$ 1986–2006
CBD 1	−0.836*** (0.052)	0.363*** (0.041)
CBD 2	−0.560*** (0.034)	0.239*** (0.028)
CBD 3	−0.455*** (0.036)	0.163*** (0.031)
CBD 4	−0.423*** (0.026)	0.140*** (0.021)
CBD 5	−0.418*** (0.032)	0.177*** (0.032)
CBD 6	−0.349*** (0.025)	0.100*** (0.024)
Counterfactuals	Yes	Yes
Agglomeration Effects	Yes	Yes
Observations	6,260	7,050
$R^2$	0.11	0.12

- Land prices are close to RF-results
- Agglomeration economies are important!



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- **Goals of this lecture**

1. **You should understand the model structure of ARSW**
  - **Simple CD productivity and utility,**
  - **Workers and productivity interact via commuting and agglomeration economies**
  - **Inelastic land market**

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- **Goals of this lecture**

2. **You should be able to estimate ARSW model**
  - **Straightforward recursive estimation using OLS/2SLS**
  - ***OR* more advanced GMM techniques**
  - **Gravity commuting equation is key!**

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- **Goals of this lecture**

3. **You should understand the pros and cons of applying the ARSW model**
  - + **Combines a structural model with **proper empirical identification****
  - + **The model seems to **replicate** reality quite well**
  - + **It relies on data sources that are widely available**
  - + **The model estimates **spatial friction/decay parameters** directly from the data**

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- **Goals of this lecture**

3. **You should understand the pros and cons of applying the ARSW model**
  - Model structure (CD/CES) is quite restrictive and does not include **other urban frictions**
  - Is the equilibrium really **unique**?
  - The modelling of **heterogeneity** is somewhat contrived
  - What do the **location fundamentals**  $(A_i, B_i)$  capture?
  - Estimation can take a long time

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- **Yesterday:**
  1. **Spatial econometrics**
  2. **Discrete choice**
  3. **Identification**
  
- **Today:**
  - ~~4. Hedonic pricing~~
  5. **Quantitative spatial economics**

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- **Yesterday:**
  1. **Spatial econometrics**
    - **Spatial data, autocorrelation, spatial regressions**
  2. **Discrete choice**
    - **Random utility framework, estimating binary and multinomial regression models**
  3. **Identification**
    - **Research design, IV, OLS, RDD, Quasi-experiments**
  
- **Today:**
  - ~~4. Hedonic pricing~~
    - ~~• Theory and estimation~~
  5. **Quantitative spatial economics**
    - **General equilibrium models in spatial economics**

# Quantitative spatial economics

Applied Econometrics for Spatial Economics

**Hans Koster**

*Professor of Urban Economics and Real Estate*