Spatial econometrics (1)

Applied Econometrics for Spatial Economics

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1. Introduction

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  - Lectures
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- Materials
  - All course materials, lecture slides, etc. can be accessed via www.urbaneconomics.nl/aese
  - If there is anything unclear, let me know!

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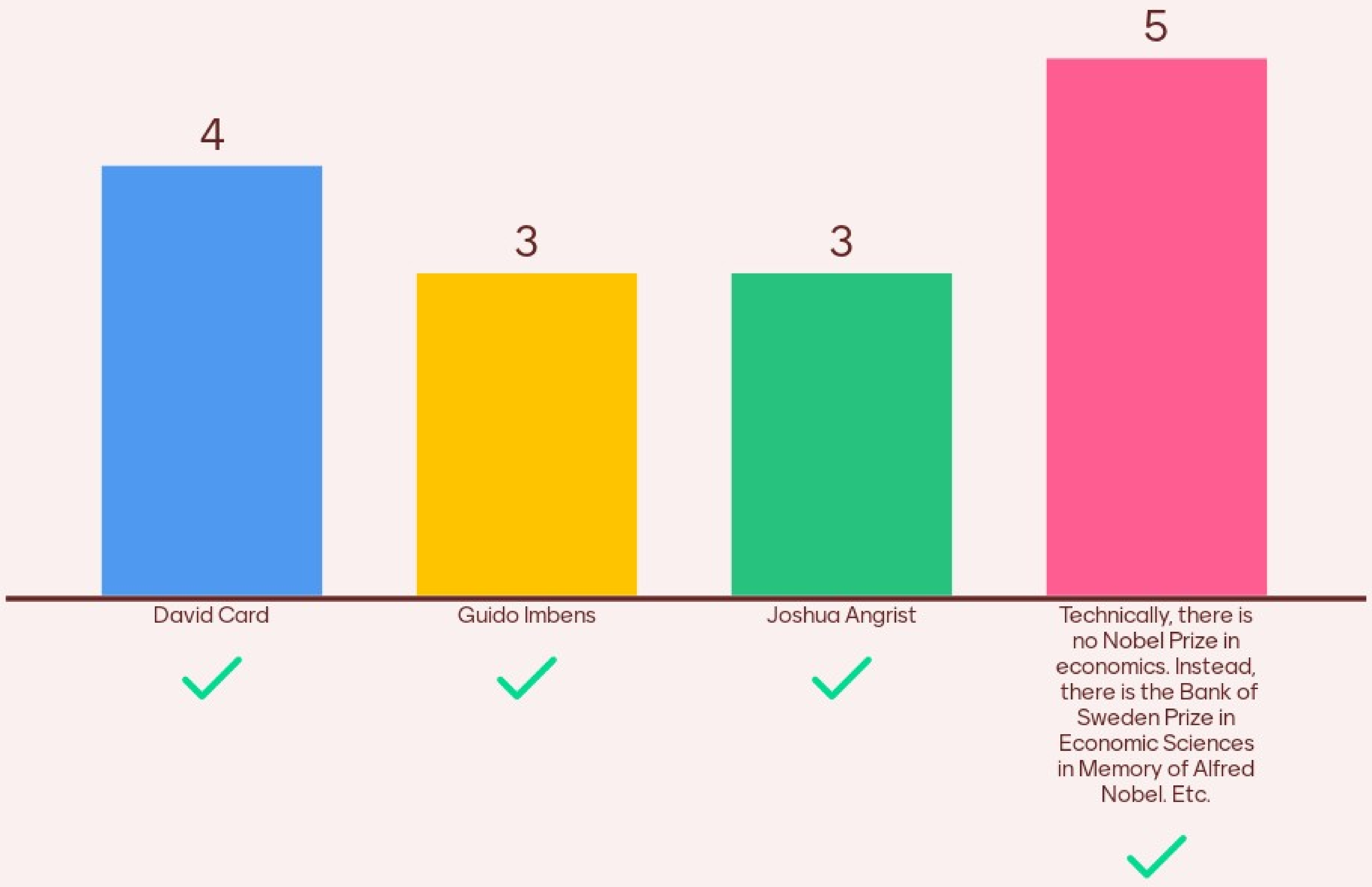
1. Introduction

- This course
  - Learn about advanced tools and techniques important for spatial economics
    ➔ No theory – an applied course!

- Do not hesitate to ask questions during the class!

- Notation on slides
  - Most important concept are underlined
  - Questions (via Menti), exercises and applications
    → On red slides

Test question: Who won the nobel prize in Economics in 2021? *(multiple answers may be correct)*



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1. Introduction

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2021

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David Card

Ill. Niklas Elmehed © Nobel Prize Outreach.
Joshua D. Angrist

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Guido W. Imbens

"for his empirical contributions to labour economics"

"for their methodological contributions to <u>the analysis of causal relationships</u>."

October 11, 2021

1. Introduction

- Today:
  1. Spatial econometrics
  2. Discrete choice
  3. Identification

- Tomorrow:
  4. Hedonic pricing
  5. Quantitative spatial economics

1. Introduction

- Today:
  1. Spatial econometrics
     - Spatial data, autocorrelation, spatial regressions
  2. Discrete choice
     - Random utility framework, estimating binary and multinomial regression models
  3. Identification
     - Research design, IV, OLS, RDD, Quasi-experiments

- Tomorrow:
  4. Hedonic pricing
     - Theory and estimation
  5. Quantitative spatial economics
     - General equilibrium models in spatial economics

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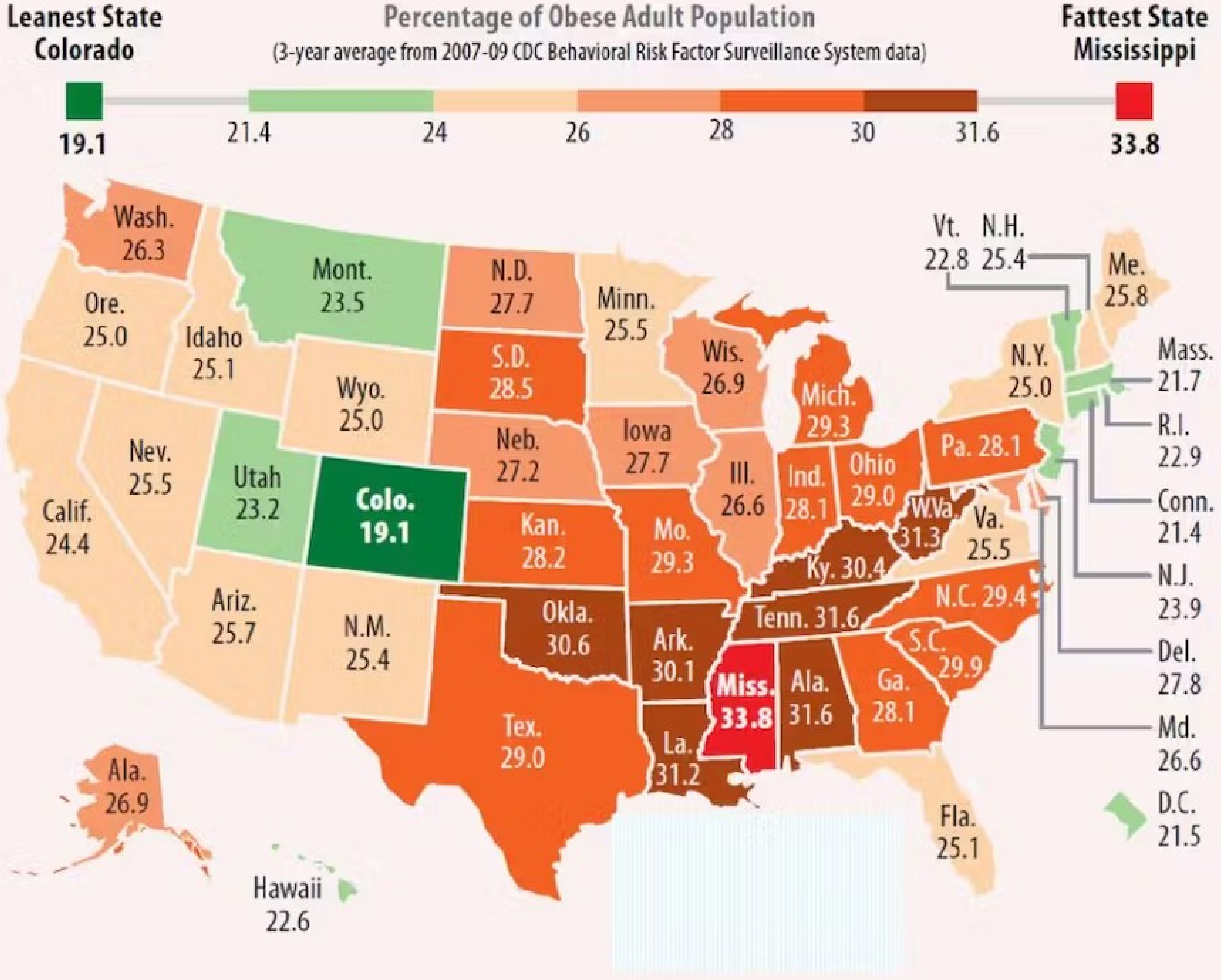
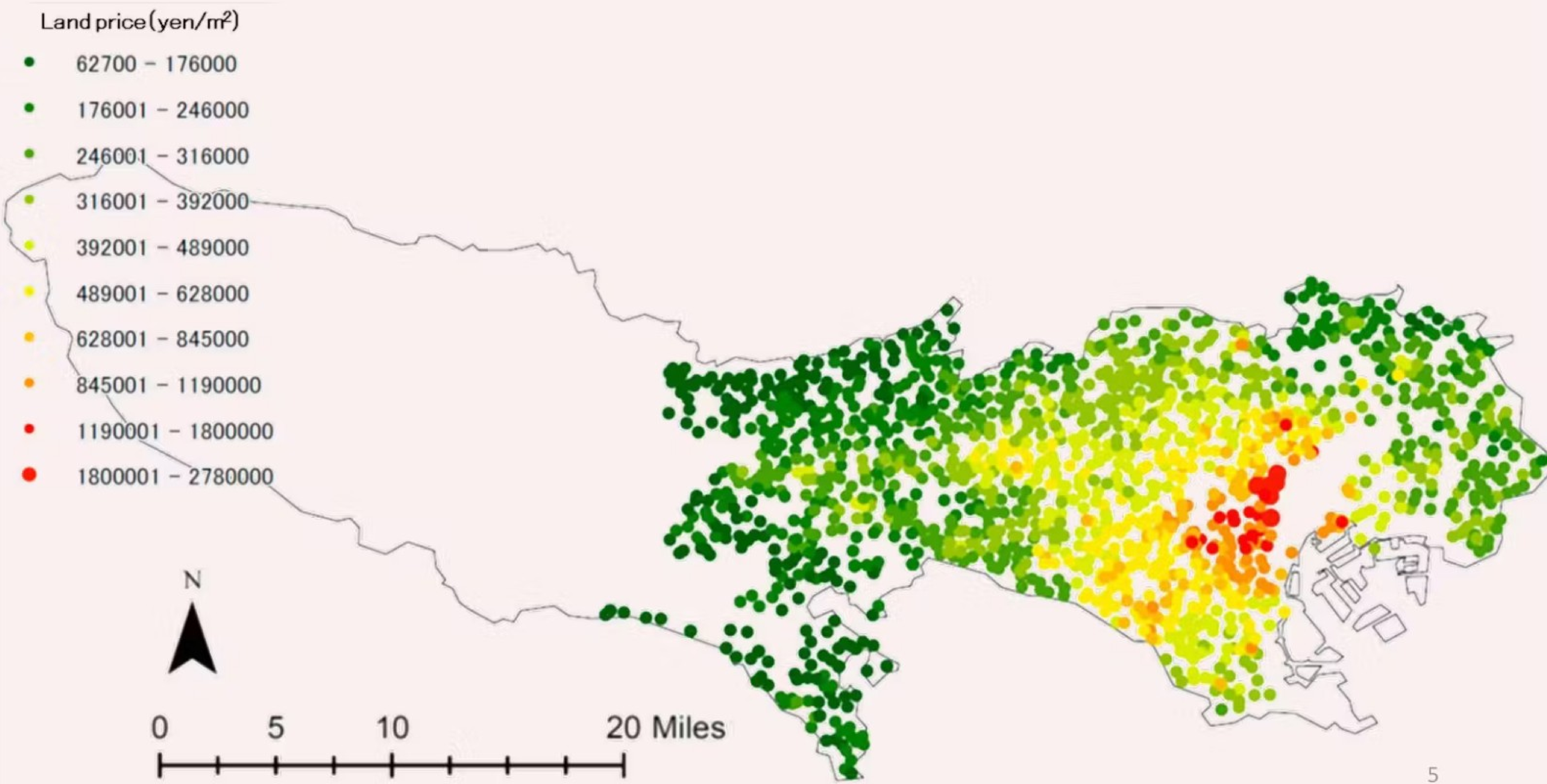
1. Introduction



- What is the m^2 price in Tokyo?



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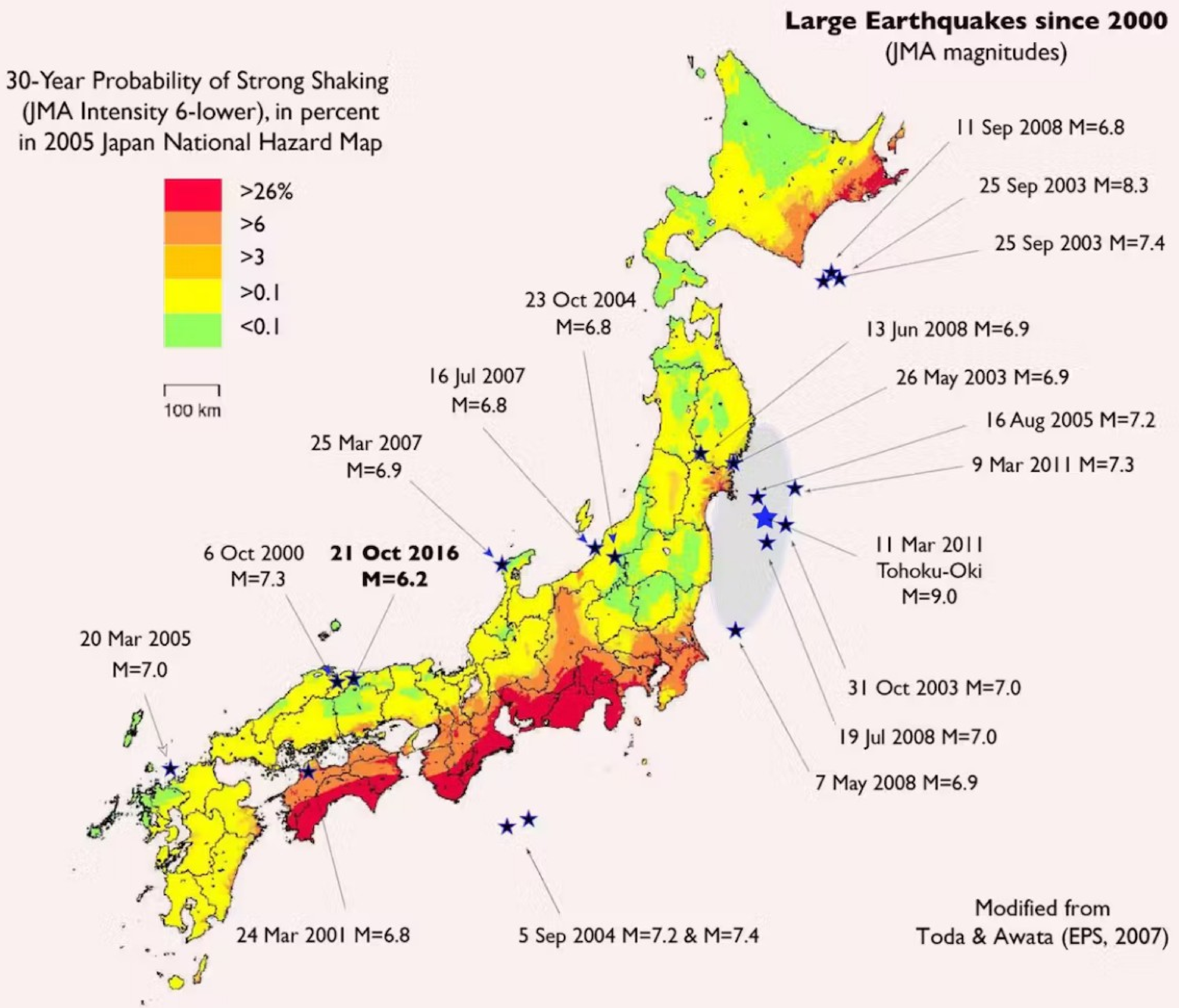
1. Introduction



- **What is special about spatial data?**

- **Not only time component, but also spatial component:**

$$y_{t,i} = \beta x_{t,i} + \epsilon_{t,i} \qquad (1')$$

- Some remarks on matrix notation
  - Use bold symbols for vectors

$$\boldsymbol{x} = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix}$$

  - Use bold symbols and capitals for matrices

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

  - Identity matrix

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow \boldsymbol{IX} = \boldsymbol{X}$

  - Inverse $\boldsymbol{X}^{-1}$ is matrix equivalent of $1/x$

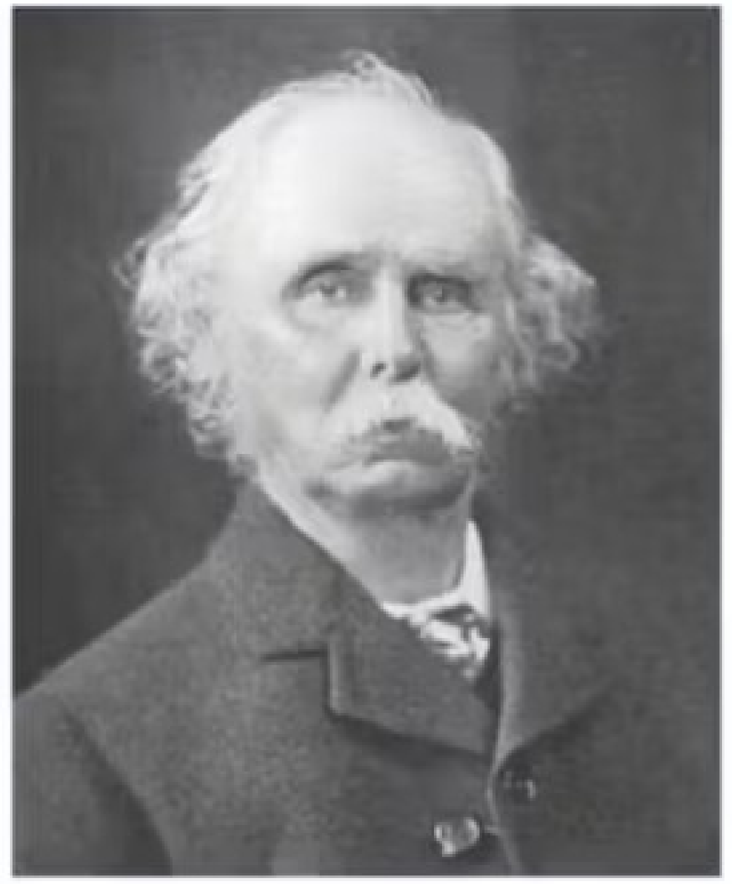
$\rightarrow \boldsymbol{X}^{-1}\boldsymbol{X} = \boldsymbol{X}\boldsymbol{X}^{-1} = \boldsymbol{I}$

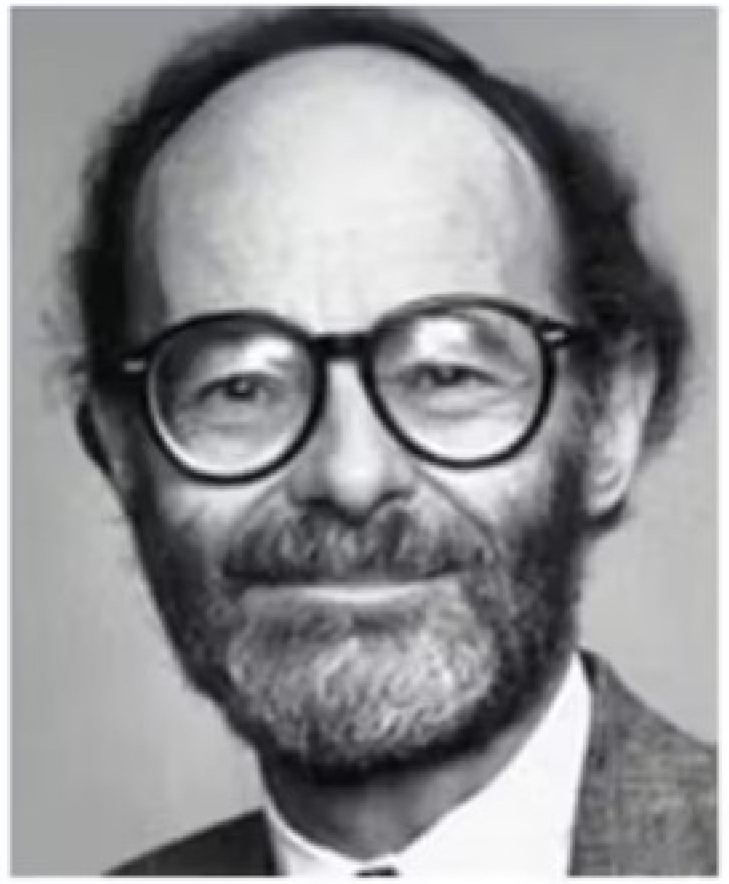
  - More details in the appendix of the syllabus

2. Space in economics

- Many economic processes are spatially correlated
  - Tobler's first law of geography
- Most economics models are "topologically invariant"
- New economic fields have emerged
  - Urban economics
  - New economic geography (NEG)
- Synergy with other fields
  - Economic geography
  - Regional science
  - GIS

- **Economists and space**





YOU?

2. Space in economics

- Spatial econometrics
- 40-50s – mainly domain of statisticians
- Cliff and Ord (1973): "Spatial autocorrelation"
- Paelinck and Klaassen (1979): "Spatial Econometrics"
- Rapid growth since Anselin (1988)
- New estimators, tests and interpretation
  - *e.g.* Kelejian and Prucha (1998, 1999, 2004, 2007, 2010)

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2. Space in economics

- Spatial modelling is becoming increasingly important
  - New and geo-referenced data
  - Advanced software
  - *New methods and regression techniques!*

3. Spatial data structure

- **Time is simple**
  - Natural origin
  - No reciprocity
  - Unidirectional

$$x_{t-3} \longrightarrow x_{t-2} \longrightarrow x_{t-1} \longrightarrow x_t$$

- **Linear space (*e.g. beach*) is different**
  - No natural origin
  - Reciprocity
  - Unidirectional

$$x_1 \longleftrightarrow x_2 \longleftrightarrow x_3 \longleftrightarrow x_4$$

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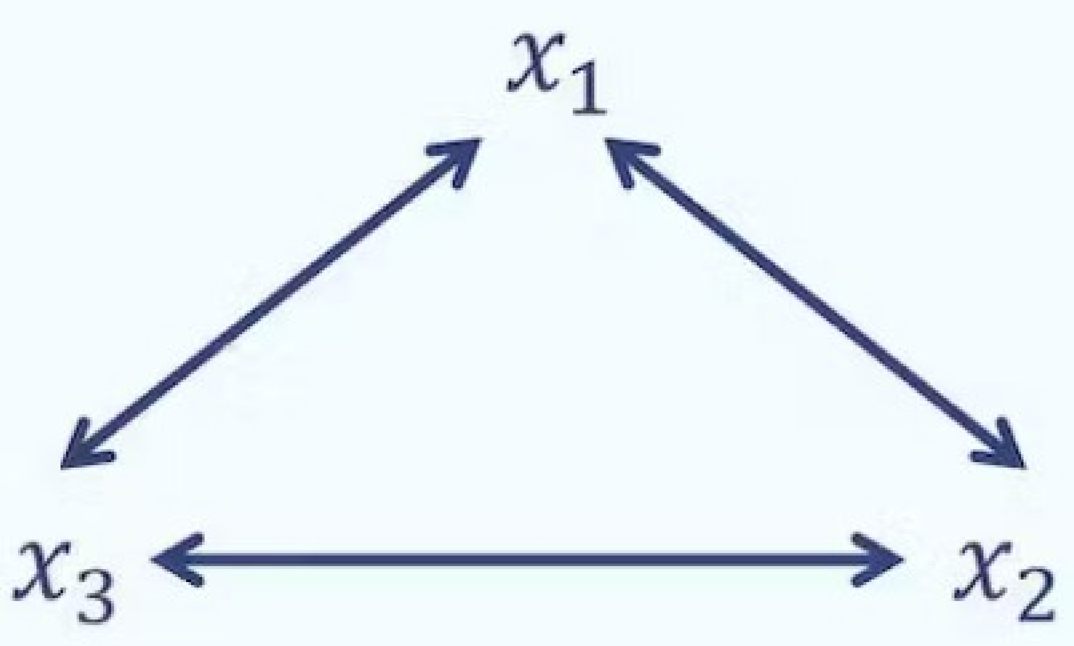
3. Spatial data structure

- **Two-dimensional space becomes even more complex**
  - **No natural origin**
  - **Reciprocity**
  - **Multidirectional**



- $i = 1,2,3$ **can refer to point data, areas, grids**

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3. Spatial data structure

- First, we have to define the spatial structure of the data

- Specified through a spatial weights matrix

- Spatial weights matrix W:
  - Consists of $n \times n$ elements
  - Discrete or continuous elements

- How to define weights?
  - Euclidian distance
  - Network distance
  - Spatial interactions
  - Social networks

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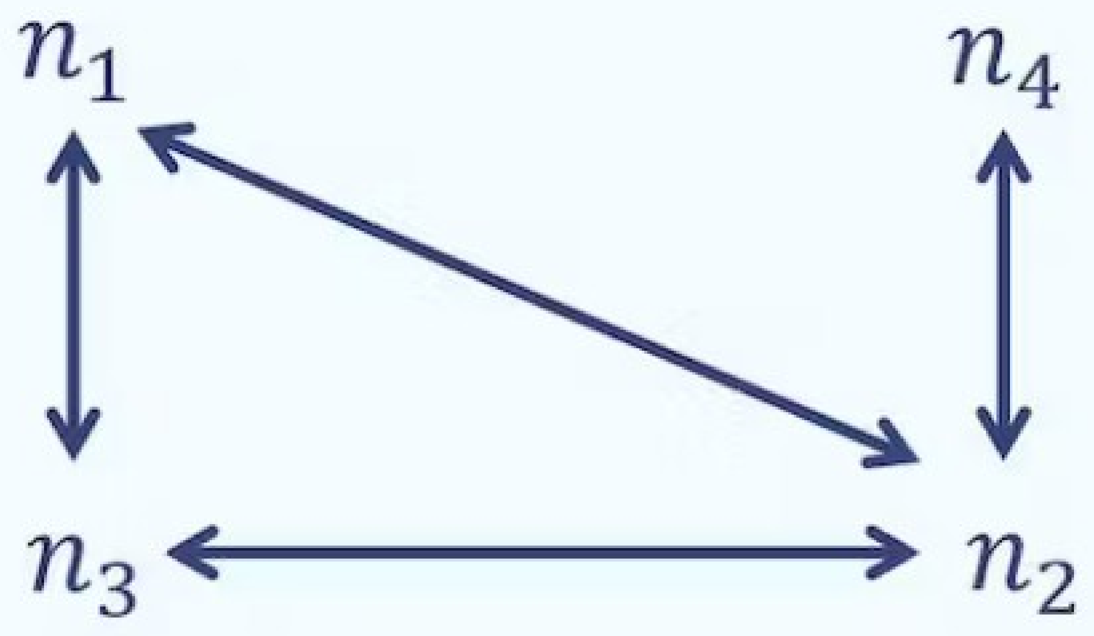
- How to define spatial matrices?

- Contiguity matrix
  - Adjacent → 1st order contiguous
  - Neighbours of neighbours → 2nd order contiguous

- Distance matrix
  - k-nearest neighbours
  - Inverse distance weights ($1/distance$)
  - Cut-off distance

- **Let's provide an example of a contiguity matrix**



to

from

W	n_1	n_2	n_3	n_4
n_1	0	1	1	0
n_2	1	0	1	1
n_3	1	1	0	0
n_4	0	1	0	0

3. Spatial data structure

- Matrices can be standardised
  - Different principles can be used
  - Most common: *row-standardisation*:

    $$w_{ij}^{*} = \frac{w_{ij}}{\sum_{k=1}^{n} w_{ik}}$$

    where k are other locations

- Interpretation of
  - $\sum_{j=1}^{n} w_{ij}$: sum of connections to neighbours
  - w_{ij}^{*} denotes the share of connections to neighbours

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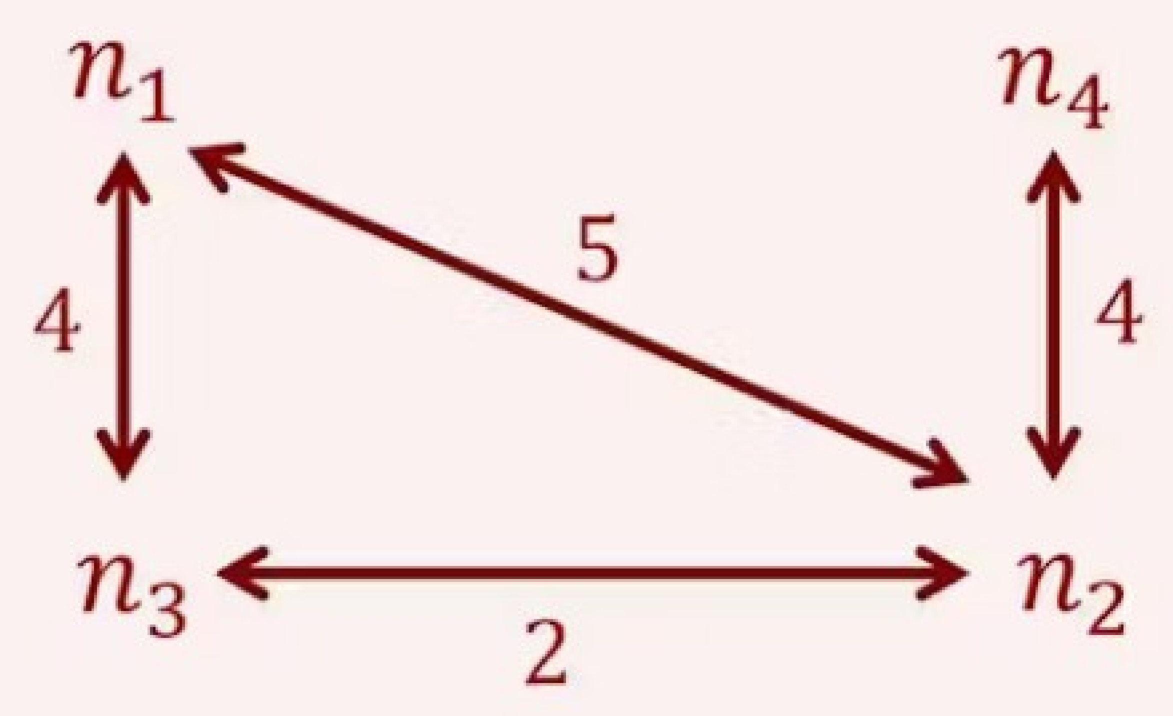
- **Create an *inverse* distance weight matrix with row-standardised weights**



to

from

W	n_1	n_2	n_3	n_4
n_1				
n_2				
n_3				
n_4				

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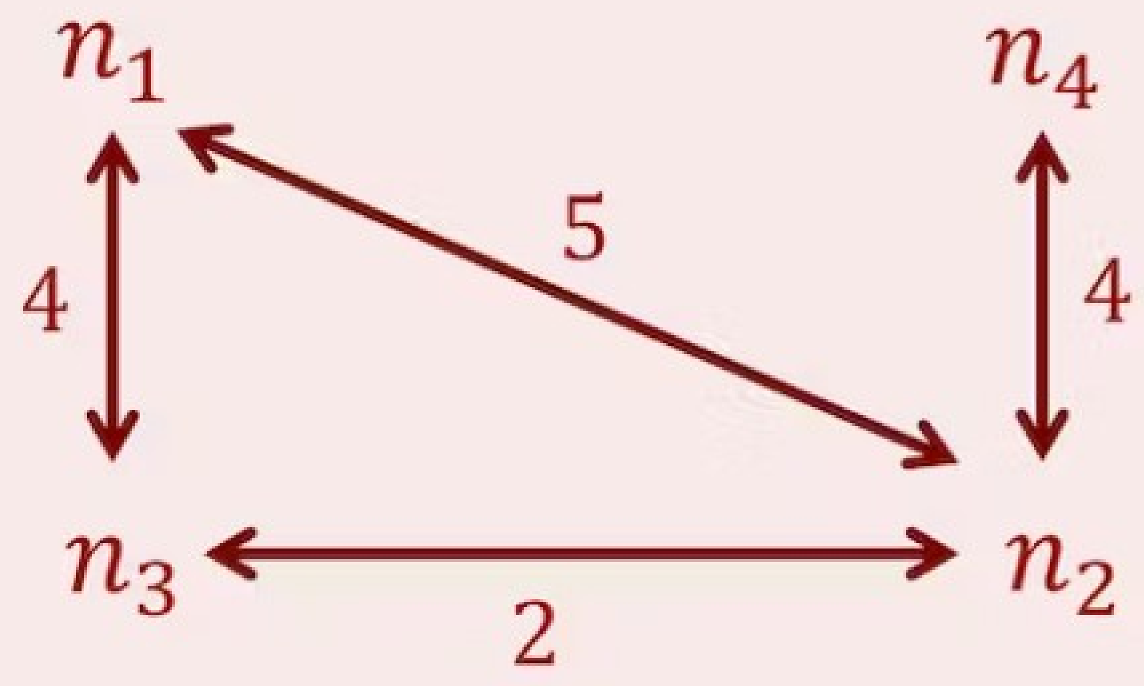
Create an inverse distance weight matrix with row-standardised weights

8

I am ready!

8

- **Create an *inverse* distance weight matrix with row-standardised weights**

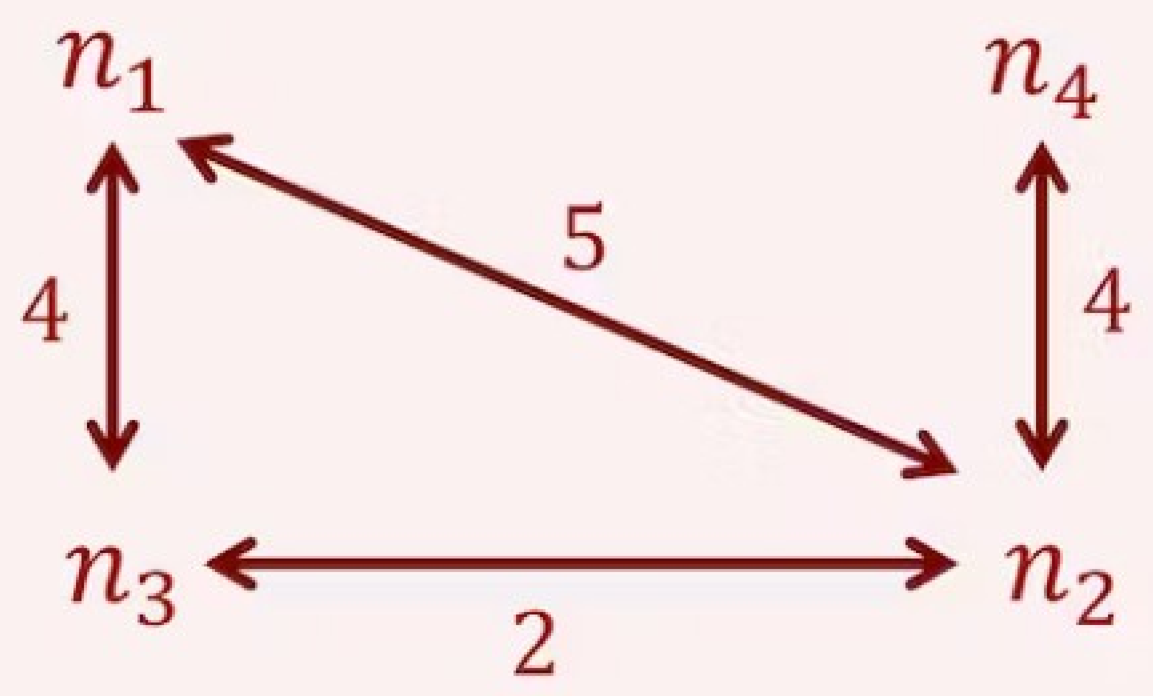


to

from

W	n_1	n_2	n_3	n_4
n_1	0	$\frac{1/5}{1/5+1/4+1/9}$	$\frac{1/4}{1/5+1/4+1/9}$	$\frac{1/9}{1/5+1/4+1/9}$
n_2	$\frac{1/5}{1/5+1/2+1/4}$	0	$\frac{1/2}{1/5+1/2+1/4}$	$\frac{1/4}{1/5+1/2+1/4}$
n_3	$\frac{1/4}{1/4+1/2+1/6}$	$\frac{1/2}{1/4+1/2+1/6}$	0	$\frac{1/6}{1/4+1/2+1/6}$
n_4	$\frac{1/9}{1/9+1/4+1/6}$	$\frac{1/4}{1/9+1/4+1/6}$	$\frac{1/6}{1/9+1/4+1/6}$	0

- **Create an *inverse* distance weight matrix with row-standardised weights**



to

from

W	n_1	n_2	n_3	n_4
n_1	0	0.36	0.45	0.20
n_2	0.21	0	0.53	0.26
n_3	0.27	0.55	0	0.18
n_4	0.21	0.47	0.32	0

- **Let's say you aim to create a spatial weight matrix**

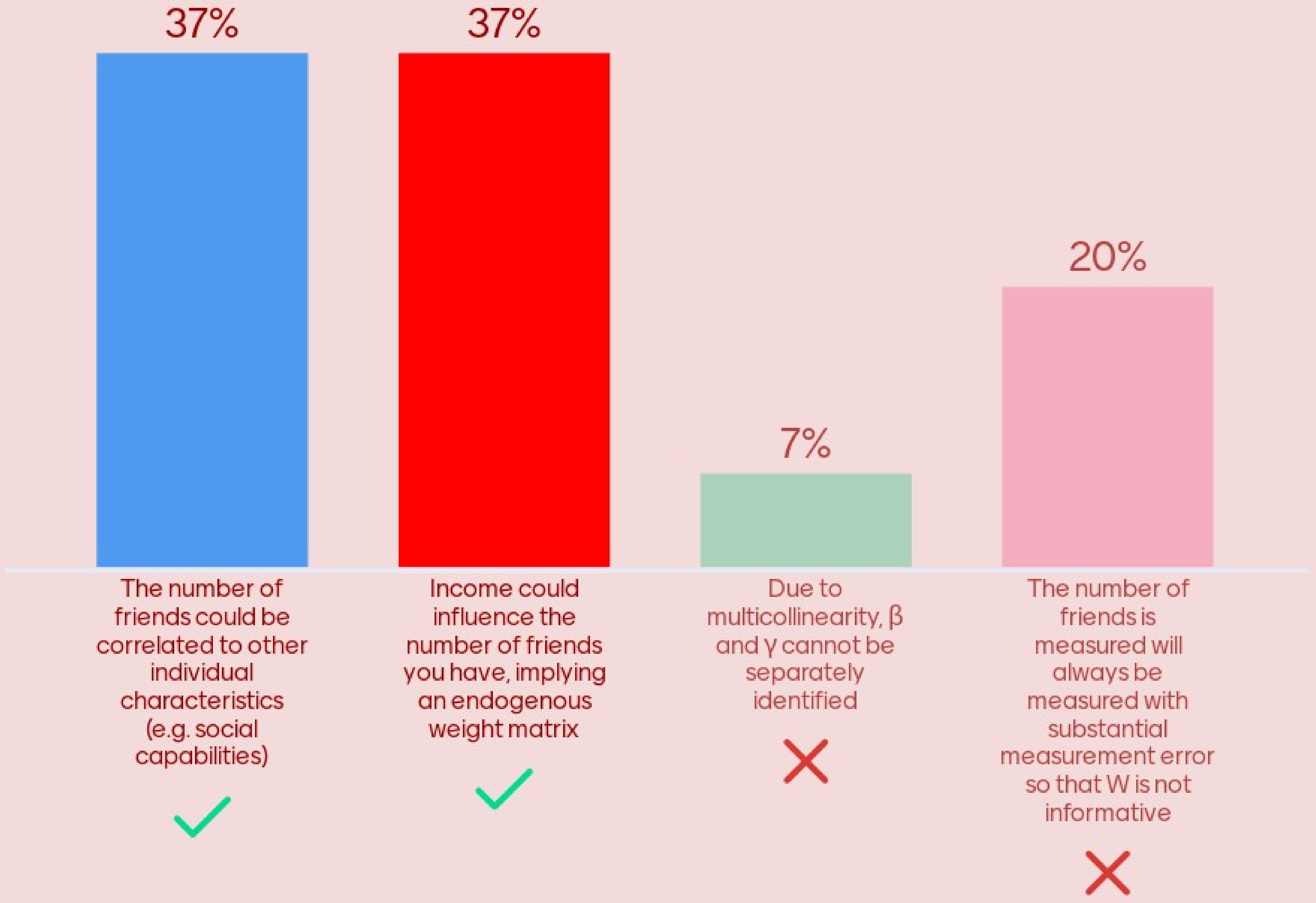
→ **What could be a problem with the following weight matrix?**

$$\boldsymbol{y} = \beta \boldsymbol{e} + \boldsymbol{W}\boldsymbol{e}'\gamma + \epsilon \quad (3)$$

$\boldsymbol{y} = \text{income}$; $\boldsymbol{e} = \text{education}$

Say that $\boldsymbol{W}$ depends on *the number of friends you have*

What could be a problem with: $\mathbf{y} = \beta\mathbf{e} + \mathbf{W}\mathbf{e}'\gamma + \varepsilon$, where $\mathbf{W}$ depends on the number of friends?

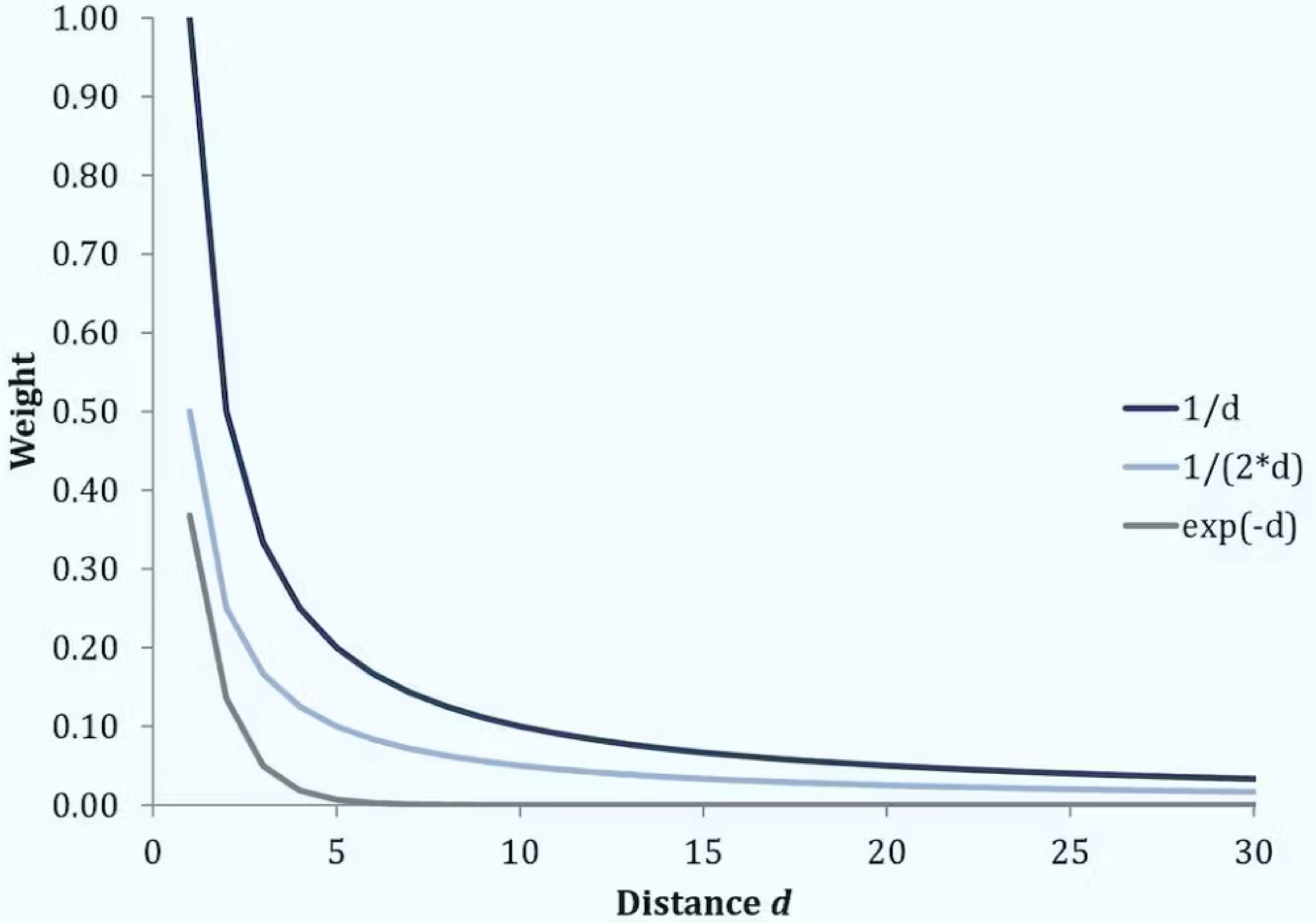


3. Spatial data structure

- Remarks regarding distance weight matrices
  - Check for exogeneity of matrix
  - Connectivity
  - Symmetry
  - Standardisation
  - Distance decay

3. Spatial data structure

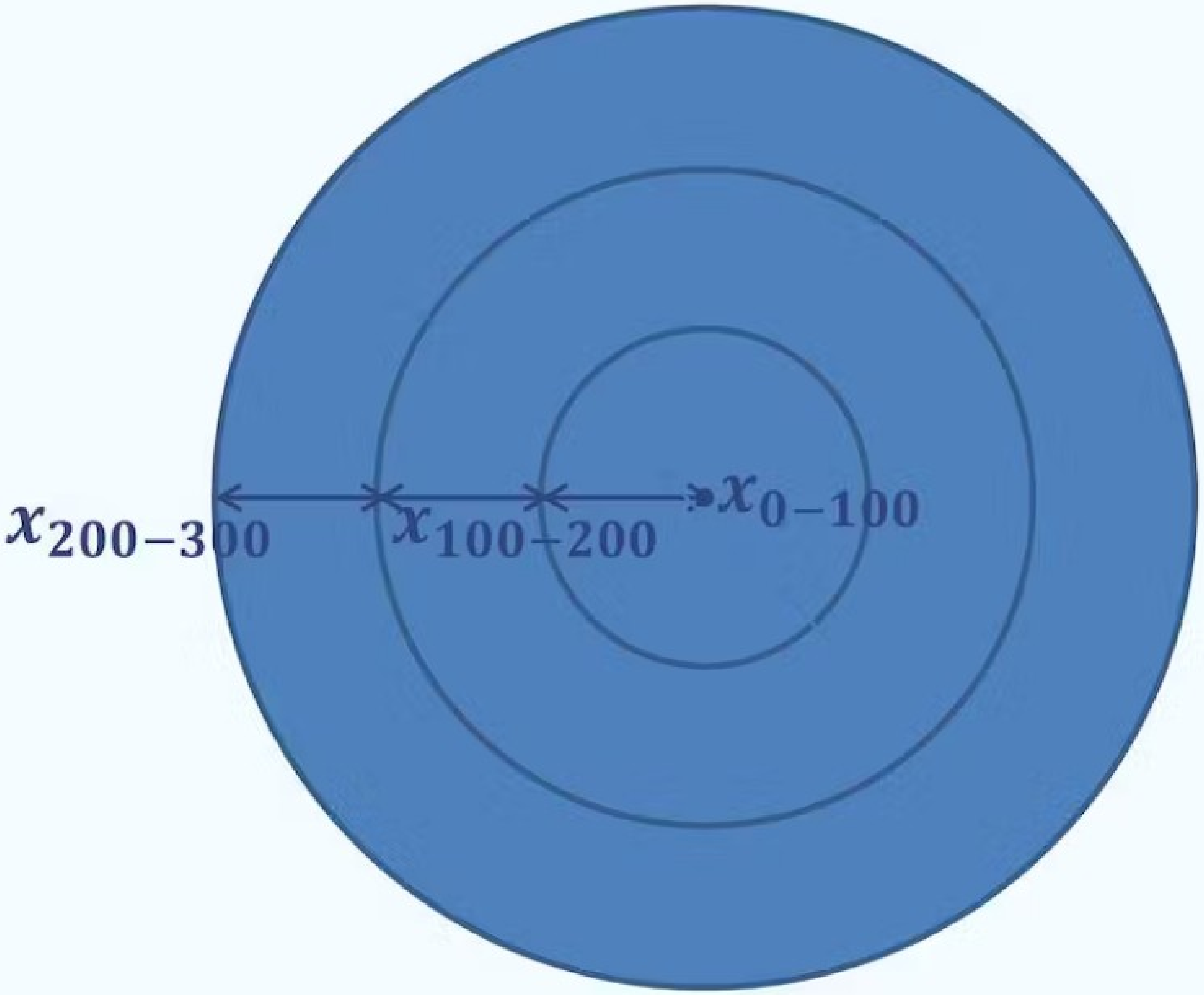
- **Choice of distance decay is arbitrary**
  - Sometimes theory may help
  - May also try to find the optimal decay parameter empirically



- Choice of distance decay is arbitrary
  - An alternative is to forget about specifying W
  - Alternatively, use different x-variables capturing concentric rings
  - Average of x-variable for different distance bands

3. Spatial data structure

- **Choice of distance decay is arbitrary**
  - e.g. $y = \alpha x_{0-100} + \beta x_{100-200} + \gamma x_{200-300} + \epsilon$



3. Spatial data structure

- **How to define spatial weight matrix using software**
  - SPATWMAT **in STATA, based on geographic coordinates**
  - SPWEIGHT **in STATA**
  - **Geoda**
  - SPATIAL STATISTICS TOOLBOX **in ArcGIS**
  - SPDEP **in *R***

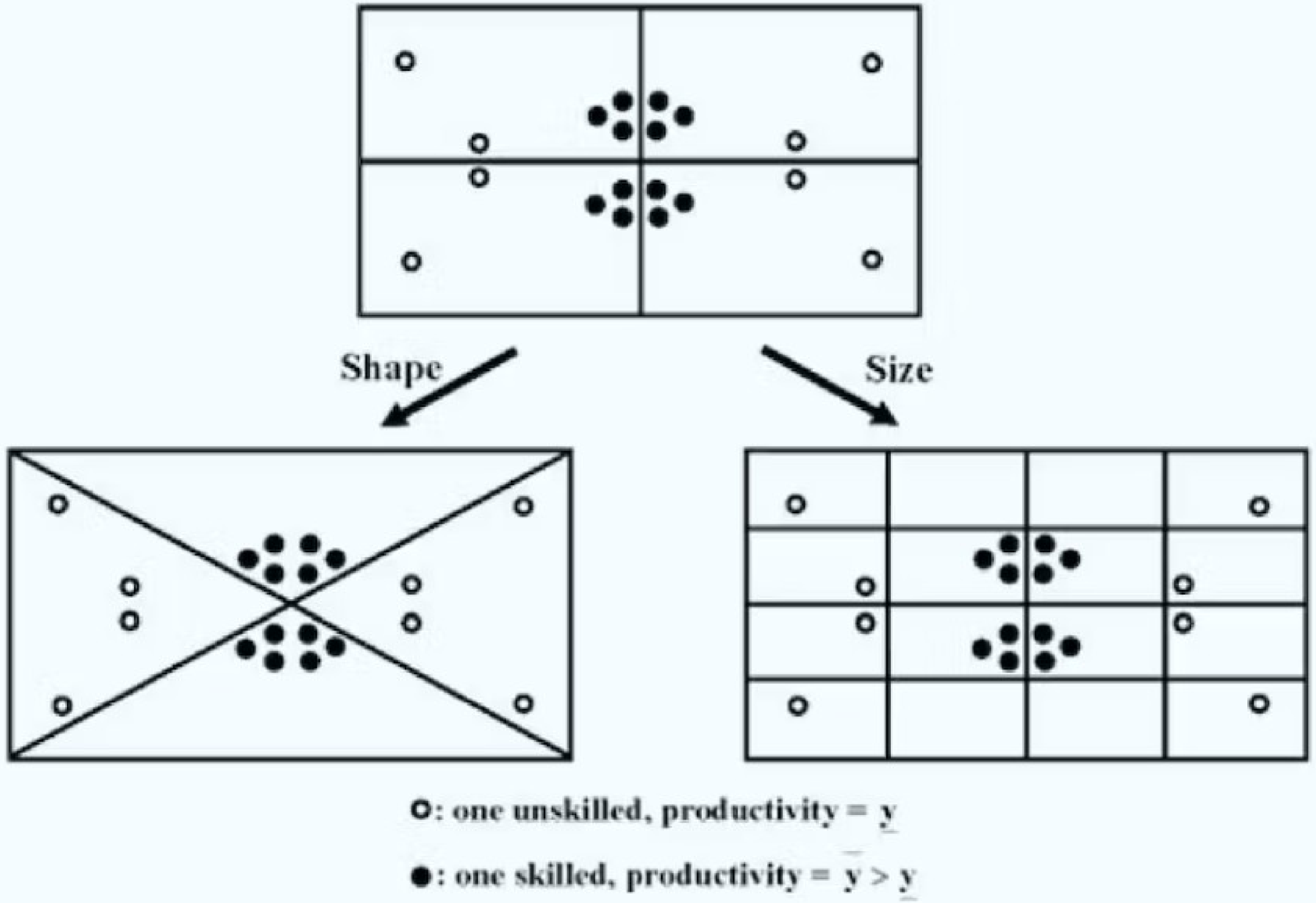
- **Concentric rings should be calculated manually**

4. Modifiable areal unit problem

- Usually we do not have space-continuous data
  - ‘Dots’ to ‘boxes’
- Data is aggregated at
  - Postcode areas
  - Municipalities
  - Regions
  - Countries
- Problems:
  - Aggregation is often arbitrary
  - Areas are not of the same size
- This may lead to distortions
  - Modifiable areal unit problem (MAUP)

4. Modifiable areal unit problem

- An illustration:

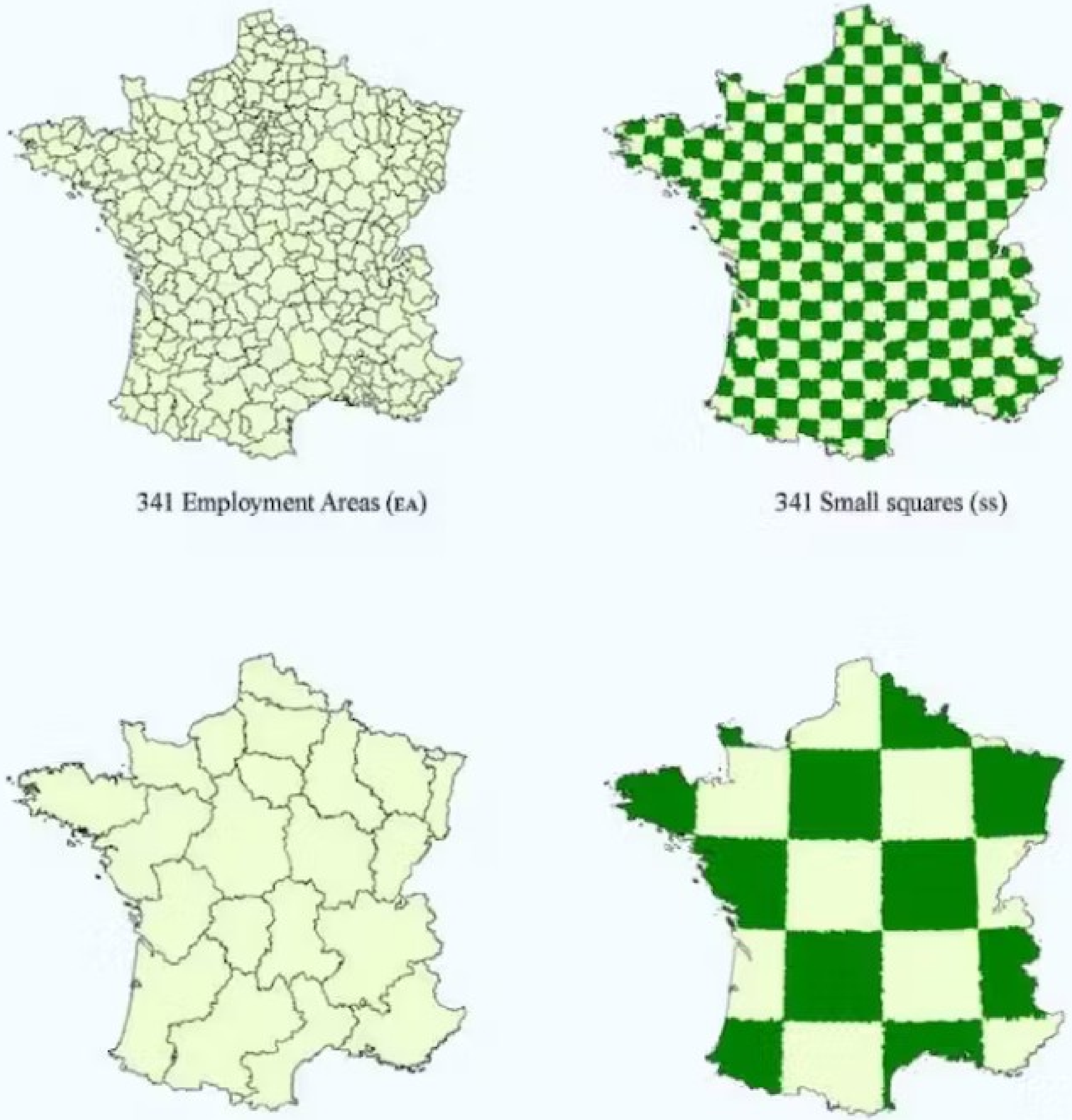


Briant, Combes and Lafourcade (2010, JUE)

- Aggregation seems to be important!

4. Modifiable areal unit problem

- **Briant et al. (2010) investigate whether choice matters for regression results**



341 Employment Areas (EA)

341 Small squares (SS)

21 Régions (RE)

22 Large squares (LS)

4. Modifiable areal unit problem

- MAUP is of secondary importance
  - If y and x are aggregated in the same way
  - Matters more for larger areas (*e.g.* regions)
  - Use meaningful areas if possible

- Specification issues are much more important

Spatial econometrics (2)

Applied Econometrics for Spatial Economics

Hans Koster

Professor of Urban Economics and Real Estate

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tinbergen institute

2. Spatial autocorrelation

- <u>Spatial autocorrelation</u> between values
  - Implies $\operatorname{cov}(x_i, x_j) = \mathrm{E}[x_i x_j] - \mathrm{E}[x_i] \cdot \mathrm{E}[x_j] \neq 0$
  - Again, j refers to other locations

- Spatial autocorrelation, dependence, clustering
  - Fuzzy definitions in literature

2. Spatial autocorrelation

- How to measure spatial autocorrelation
  - Moran's *I*
  - Focus on one variable x (e.g. crime)

- H_0: independence, spatial randomness
- H_A: dependence
  - On the basis of adjacency, distance, hierarchy

- **Moran's I is given by:**

$$I = \frac{R}{S_0} \times \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}} \qquad (4)$$

where R is the number of spatial units
S_0 is the sum of all elements of the spatial weight matrix
W is the spatial weight matrix
$\tilde{x} = x - \bar{x}$ is a vector with the variable of interest

- **Use row-standardised spatial weight matrix W!**
  - **So that $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}}$**

2. Spatial autocorrelation

- **Moran's I**

- **Recall that $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}}$ (standardised I)**
  - **Note similarity with OLS: $\hat{\beta} = \frac{x'y}{x'x}$**
  - **Hence: $W\tilde{x} = \alpha + I\tilde{x} + \epsilon$, where $\alpha = 0$**

- **Moran's I is correlation coefficient (more or less)**
  - $\approx [-1,1]$
  - **But: expectation $\mathrm{E}[I] = -\frac{1}{N-1}$**

- **Visualisation**
  - **Moran scatterplot**

2. Spatial autocorrelation

- **Moran's *I***

- **Sidenote:**
  - **Please realise that $W\tilde{x}$ is a vector**
    - $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}'\tilde{x}}$

  - $\mathbf{W} \quad \times \quad \tilde{x} \quad = \mathbf{W}\tilde{x}$

$$\begin{bmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{bmatrix} \times \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

  - **Notation:** $\frac{x'y}{x'x} = x^T y \left(x^T x\right)^{-1}$

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2. Spatial autocorrelation

- **Moran's I**

- **How to investigate the statistical significance of (4)?**
  - $\frac{I - \mathrm{E}[I]}{\sqrt{\mathrm{var}[I]}}$ **(5)**
  - **However, $\sqrt{\mathrm{var}[I]}$ is difficult to derive**
  - $\mathrm{E}[I] = -1/(n-1)$
  - **Assume normal distribution of I to approximate $\sqrt{\mathrm{var}[I]}$ under H_0**
  - **Or: bootstrapping/simulation**

  - **See Cliff and Ord (1973) for more details**

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2. Spatial autocorrelation

- **Moran's *I***

- **Also possible: correlation to other variables:**

$$I_S = \frac{\tilde{x}' W \tilde{z}}{\tilde{x}' \tilde{x}}$$

2. Spatial autocorrelation

- **How to calculate Moran's *I* using software**
  - SPAUTOC **in STATA**
  - SPLAGVAR **in STATA**
  - SPATIAL STATISTICS TOOLBOX **in ArcGIS**

- **Alternative: Getis and Ord's *G***
  - **Most of the time only Moran's *I* is reported**

- **Let's try to answer the queston:**
  "Is social deprivation spatially clustered?"

- **How to determine the most deprived neighbourhoods?**

- **Dutch government calculated deprivation *z*-score for each neighbourhood**
  - **Based on housing quality, safety, perception and satisfaction**
  - ***Important:* the 83 most deprived neighbourhoods were selected for an investment of >€1 billion**

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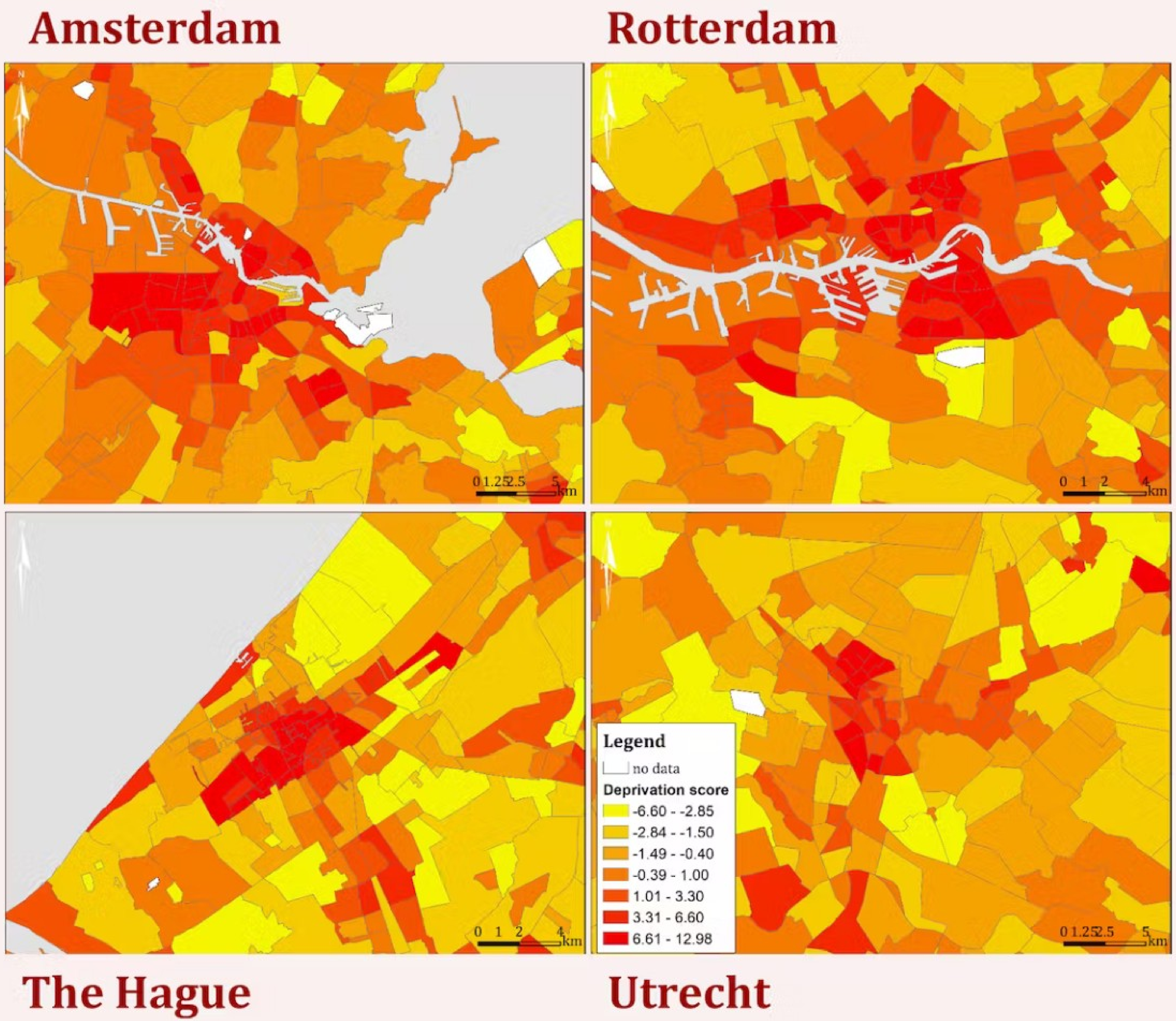
1. Introduction
2. Spatial autocorrelation
3. Spatial regressions
4. Summary



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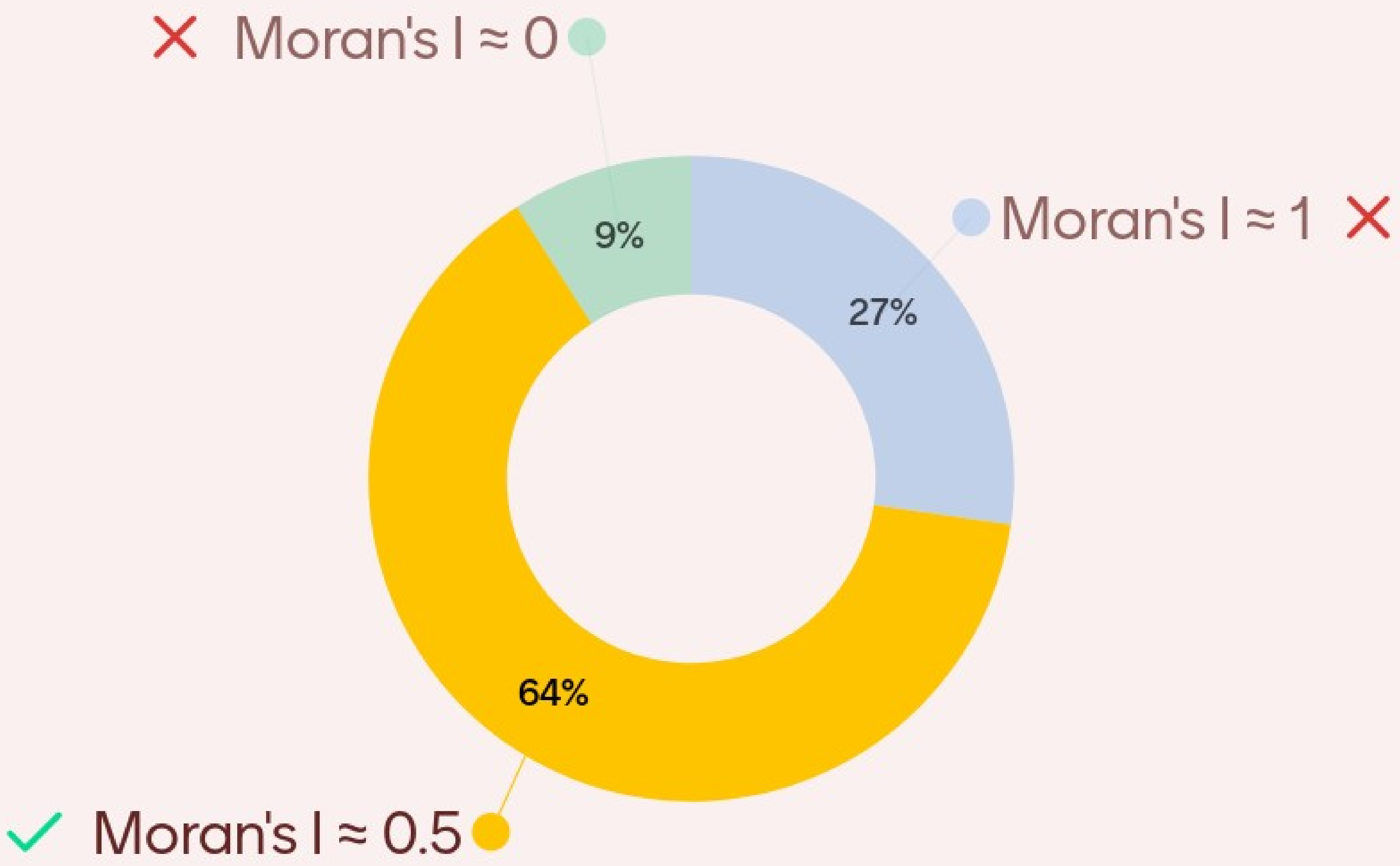
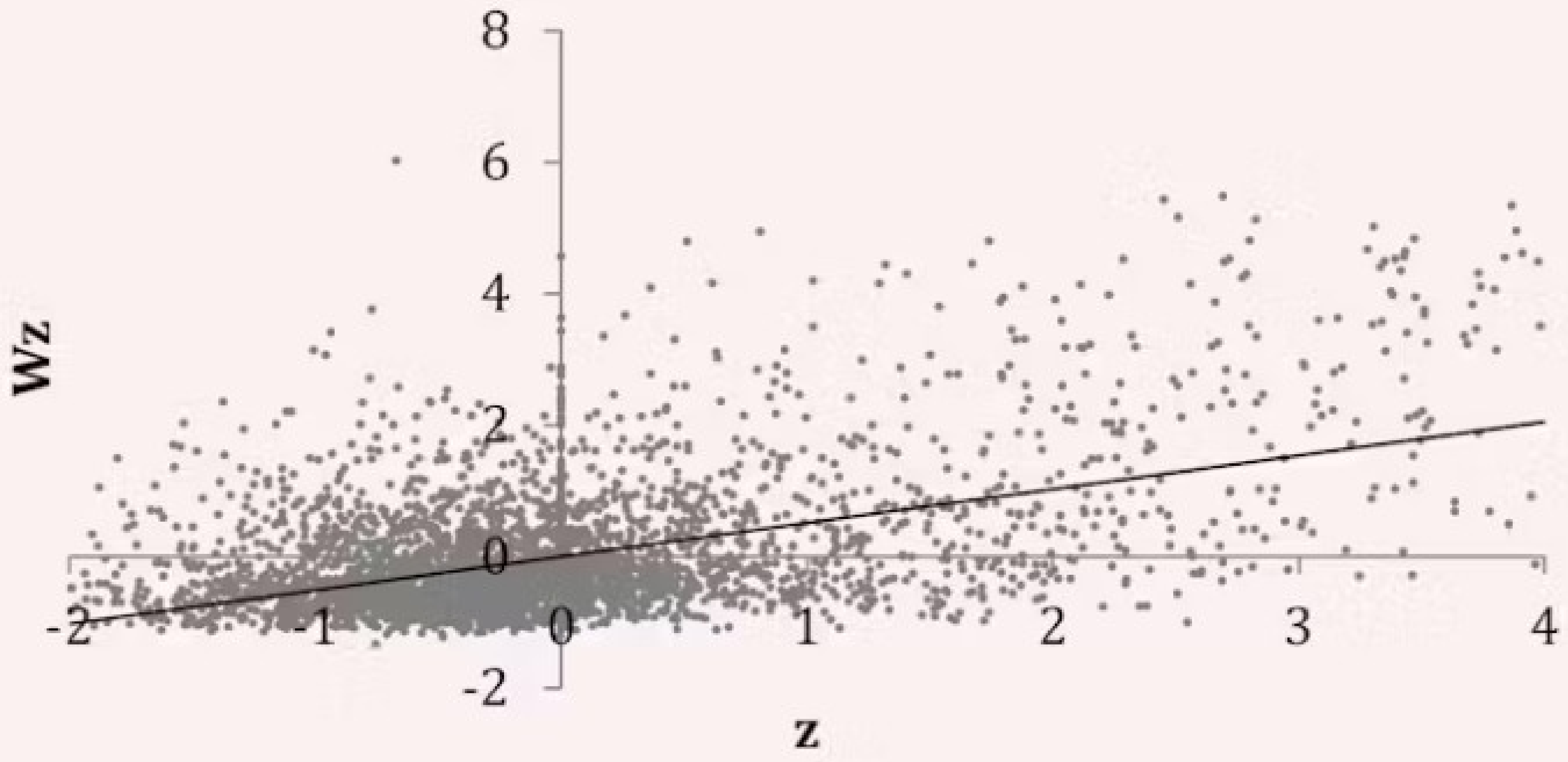
- Determine spatial autocorrelation
  1. Determine distance between all neighbourhoods using centroids
  2. Use inverse distance function $w_{ij} = 1/\left(d_{ij}^{\gamma}\right)$ to determine spatial weights in weight matrix
  3. Calculate Moran's I: $\mathbf{W}\tilde{\mathbf{z}} = \alpha + I\tilde{\mathbf{z}} + \epsilon$ where $\tilde{\mathbf{z}} = \mathbf{z} - \bar{z}$ and W is a row-standardised weight matrix
     - *Recall that* $\mathbf{W}\tilde{\mathbf{z}}$ *is a vector*
  4. Bootstrap this procedure to estimate standard error (or use software)

What is your hypothesis when looking at the figure

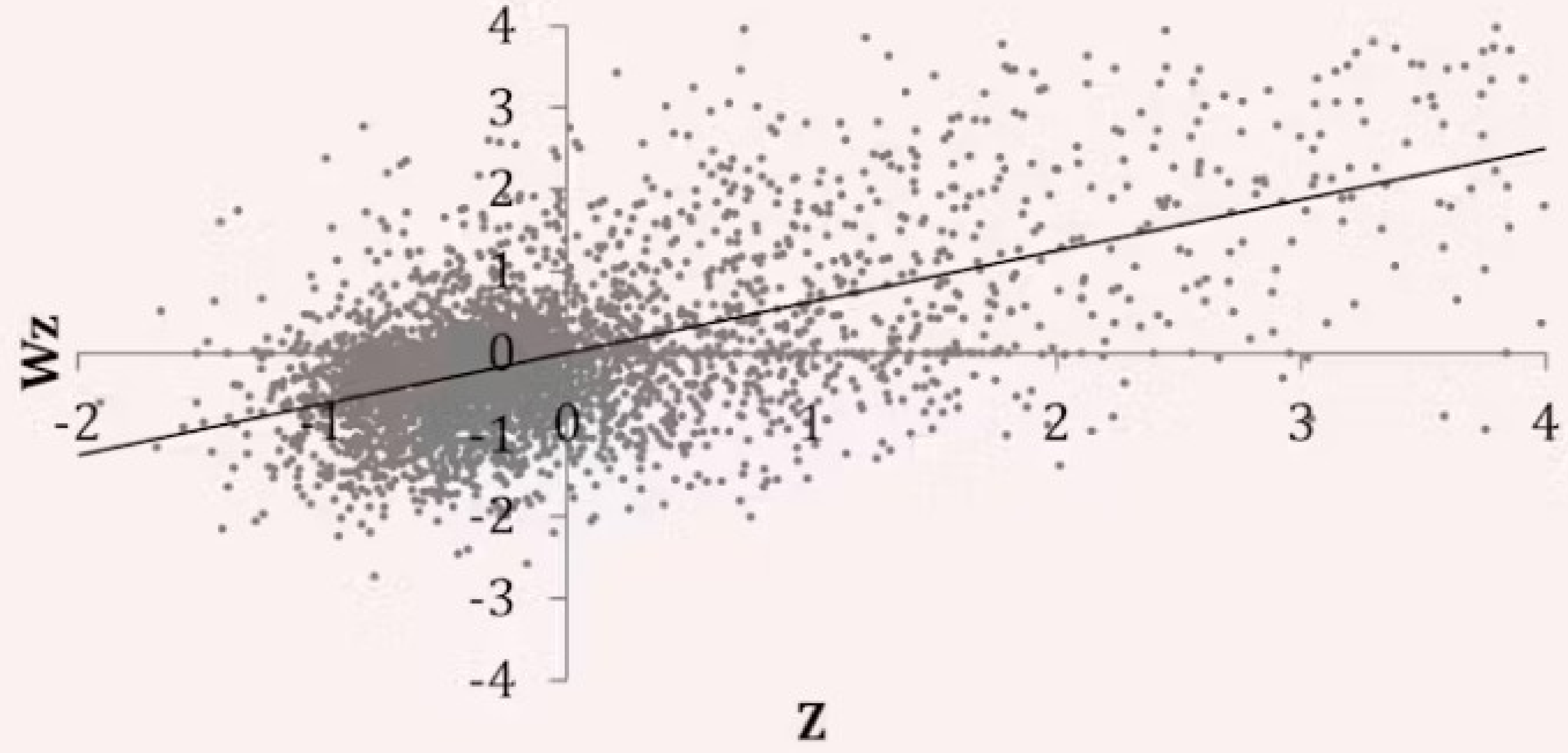


- **Calculate Moran's I**
  - **Using inverse distance function** $w_{ij} = \frac{1}{d_{ij}^{\gamma}}$

I = 0.513*** *(γ=1)*



I = 0.625*** *(γ=2)*



- Spatial correlation in deprivation
  - Local phenomenon?
  - You do not know *why* scores are autocorrelated...
  - No causal relationships!

2. Spatial autocorrelation

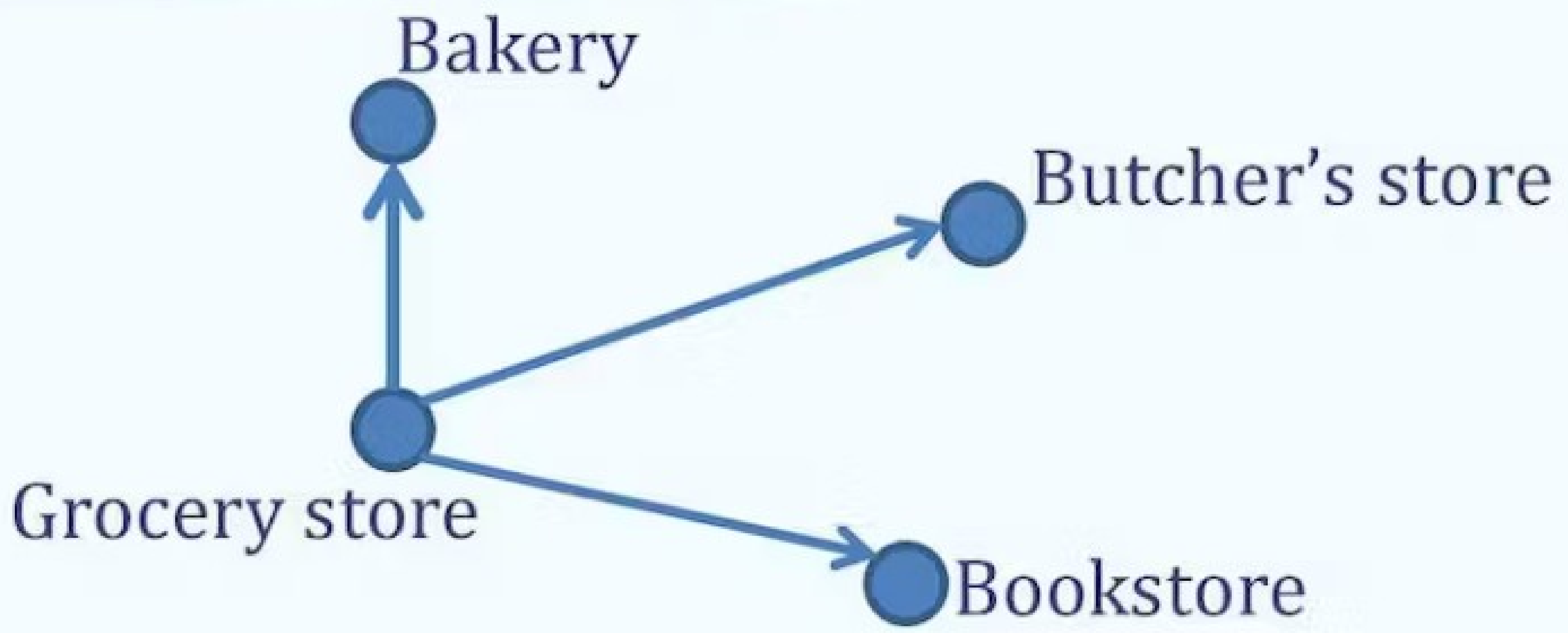
- It is important to make a distinction between *global* and *local* spatial autocorrelation
  - See Anselin (2003) for a discussion

- Global spatial autocorrelation
  - Local shock affects the whole system

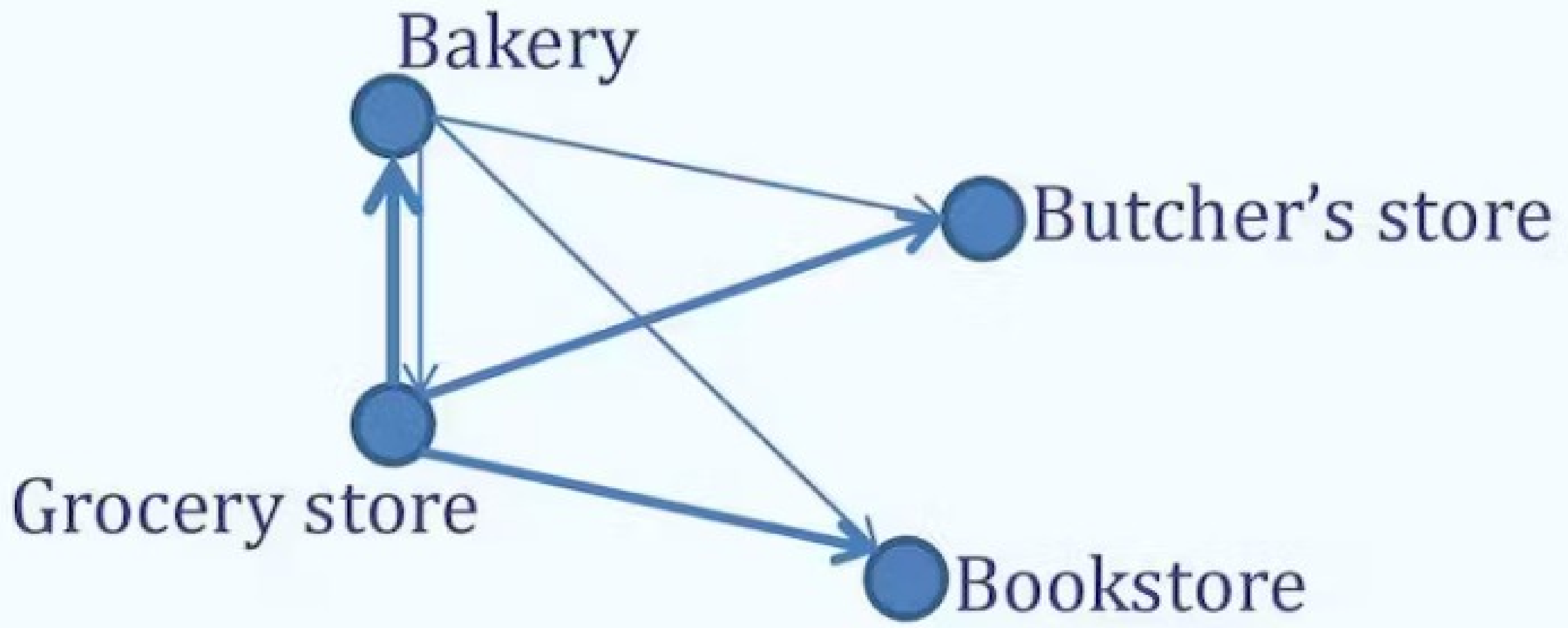
- Local spatial autocorrelation
  - Local shock only affects the 'neighbours'

2. Spatial autocorrelation

- **Example: Consider an income increase for grocery store owner**
- **Local autocorrelation:**

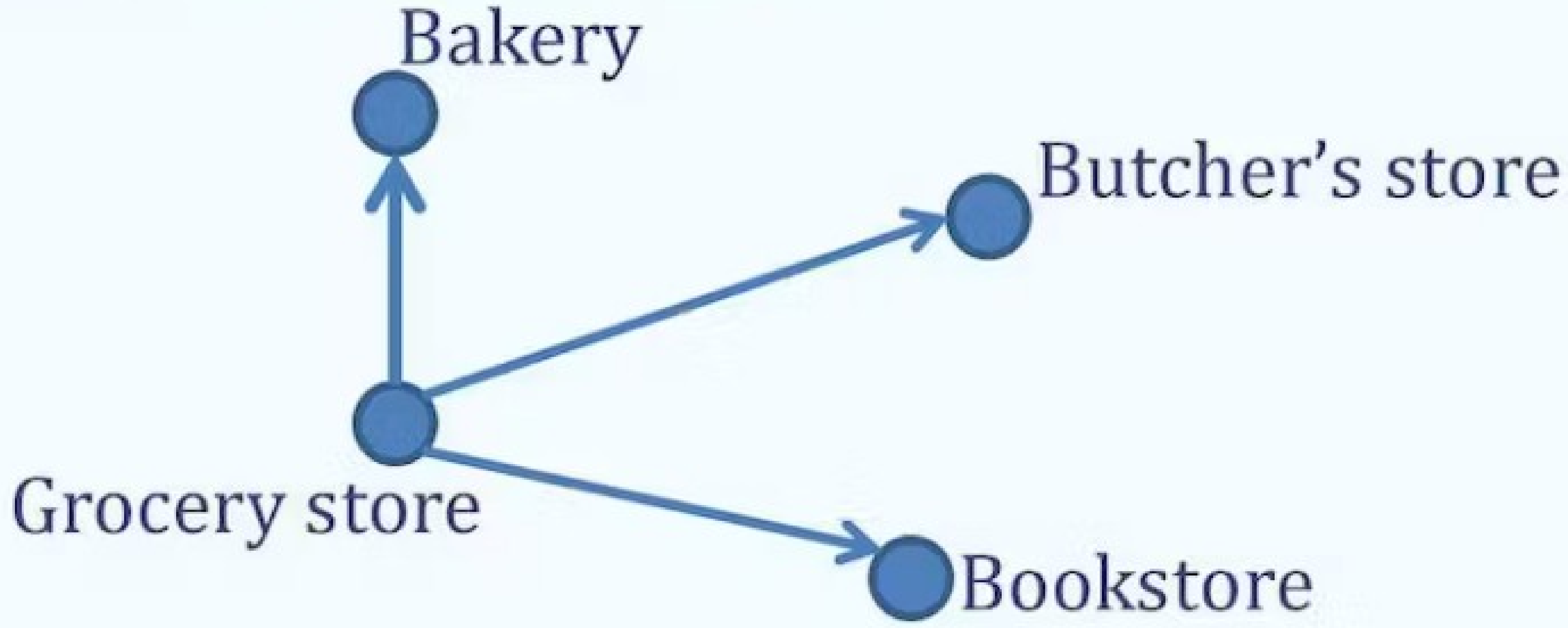


- **Global autocorrelation:**

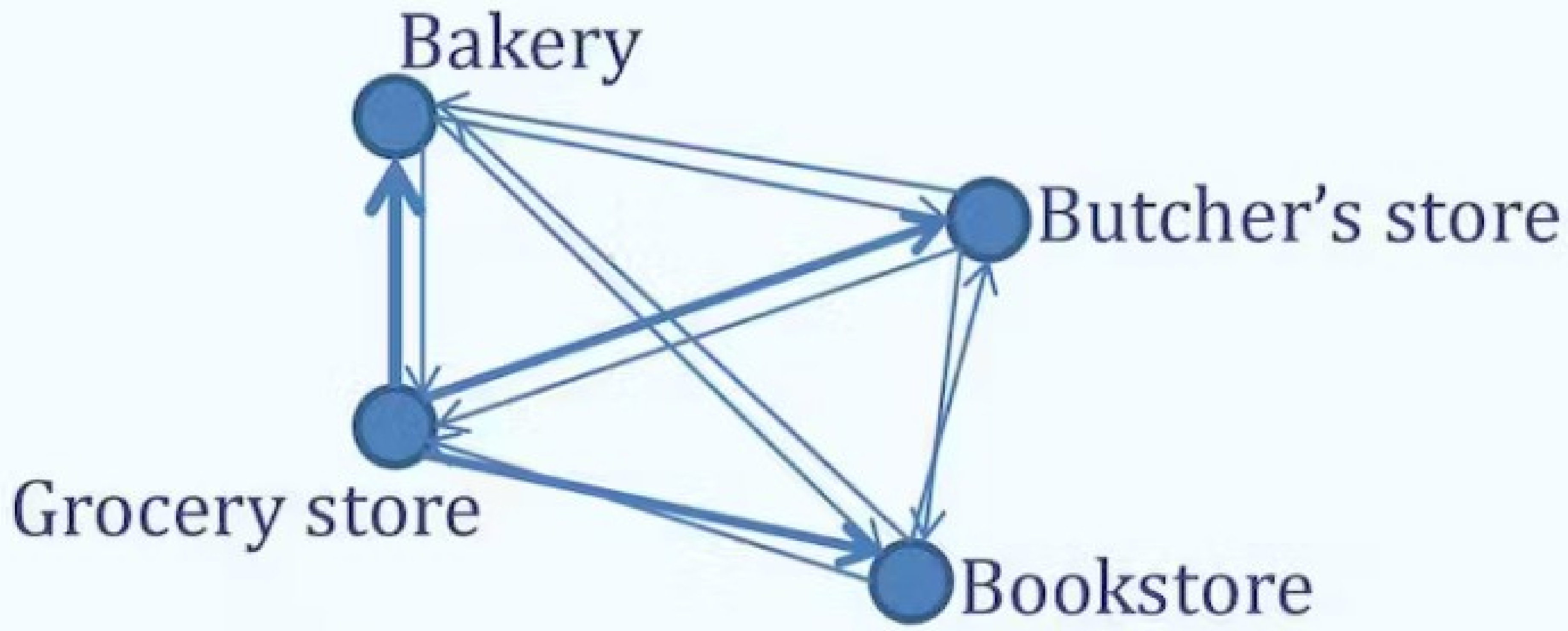


2. Spatial autocorrelation

- **Example: Consider an income increase for grocery store owner**
- **Local autocorrelation:**



- **Global autocorrelation:**



… spatial multiplier process

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2. Spatial autocorrelation

- Let's define $z = \lambda W z + \mu$
  - Reduced-form of z is $z = [I - \lambda W]^{-1}\mu$
  - With $\lambda < 1$

- A Leontief expansion yields:
  - $[I - \lambda W]^{-1} = I + \lambda W + \lambda^2 W^2 + \lambda^3 W^3 + \cdots$

- $W^2 \rightarrow$ There is an impact of neighbours of neighbours (as defined in W) although it is smaller (λ^2)
  - Global autocorrelation
  - *Spatial multiplier* process
  - In practice: covariance may approach zero after a relatively small number of powers

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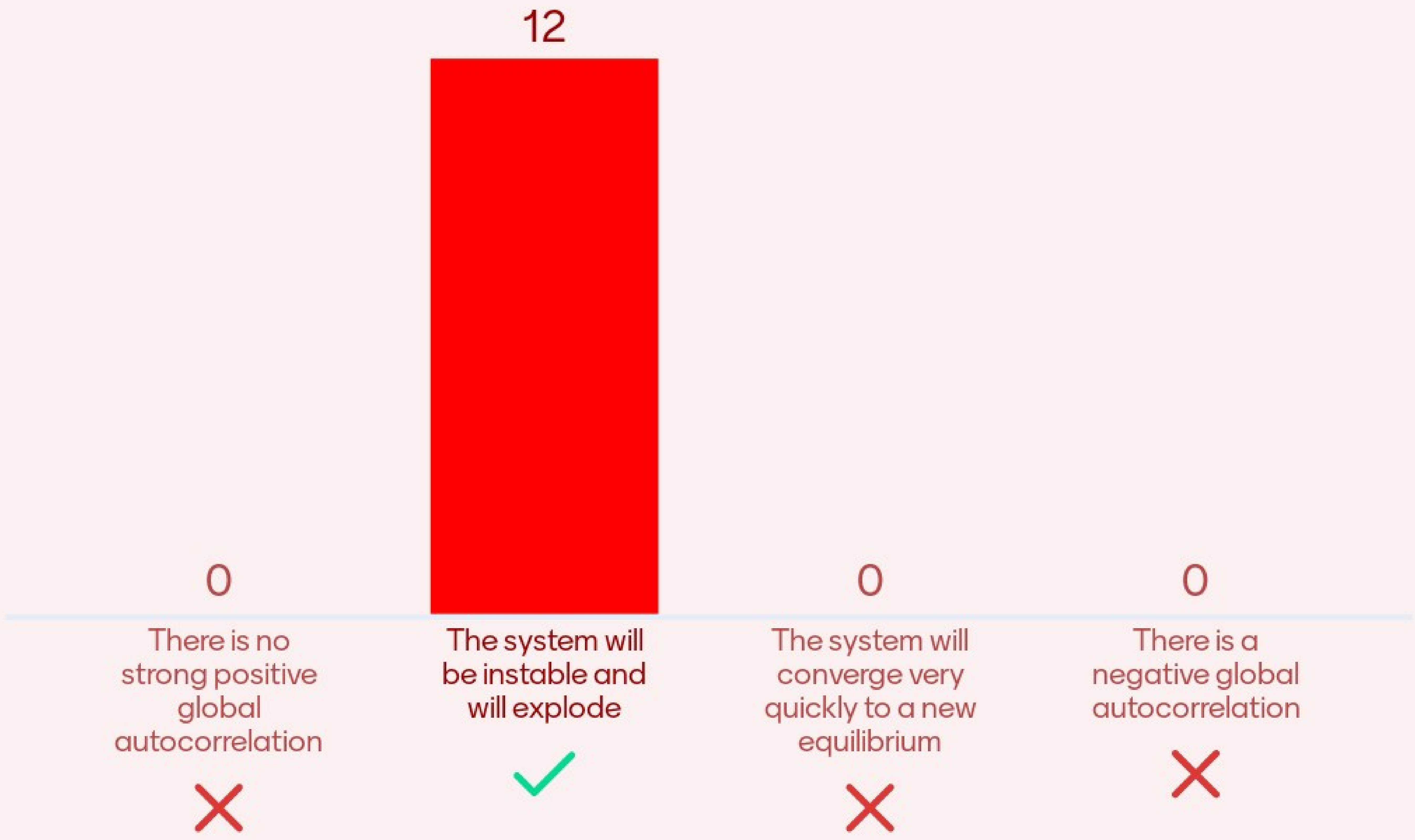
What happens when $\lambda > 1$ in $\mathbf{z} = \lambda \mathbf{W} \mathbf{z} + \mu$?



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2. Spatial autocorrelation

- Let's define $z = \lambda W \mu + \mu$
  - This is already a reduced-form of z
- No impact of behaviour beyond 'bands' of neighbours
  - Dependent on definition of W
  - …Local autocorrelation
- Covariance is zero beyond these bands

2. Spatial autocorrelation

- Local or global autocorrelation?
  - Dependent on application
  - Theory...

- **Taxonomy:**

$$y = \rho W y + X\beta + W X \gamma + \epsilon \qquad (1)$$

with

$$\epsilon = \lambda W \epsilon + \mu \qquad (2)$$

W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

3. Spatial regressions

- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu}$ (3)
  - $\rho \neq 0, \boldsymbol{\gamma} = 0, \lambda = 0$
  - Spatial dependence in dependent variables

- **Note similarity with time-series models**
  - AR Model
  - $\boldsymbol{y}_t = \rho \boldsymbol{y}_{t-1} + \boldsymbol{X}_t\boldsymbol{\beta} + \boldsymbol{\mu}_t$ (4)

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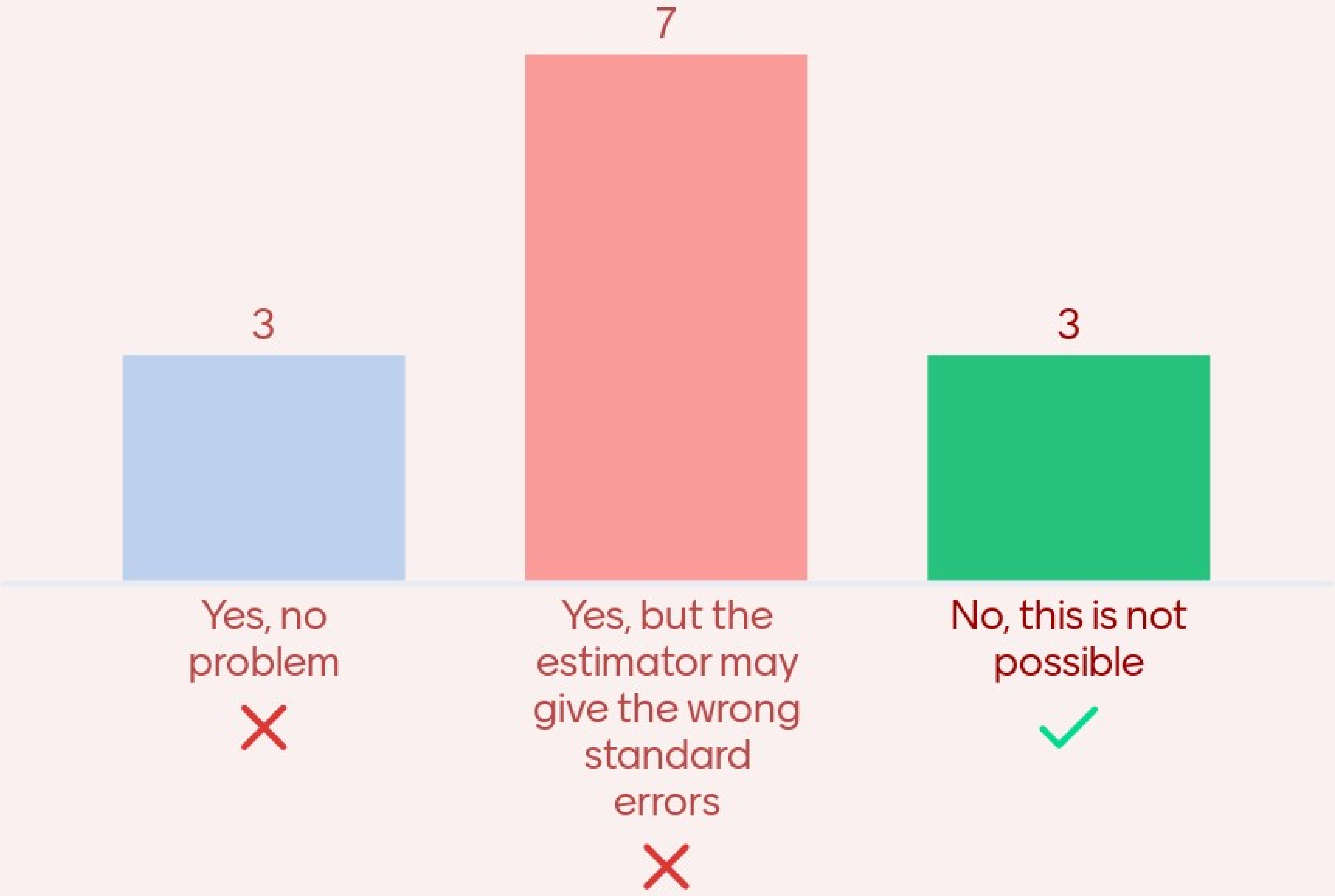
- **Spatial lag model**
  - $y = \rho W y + X\beta + \mu$ (3)

- **The outcome variable influences everyone (indirectly)**
  - **Global autocorrelation**

- **We may write**
  $(\mathbf{I} - \rho W)y = X\beta + \epsilon$
  $y = (I - \rho W)^{-1}(X\beta + \mu)$ **with**
  $(I - \rho \mathrm{W})^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \cdots$

Can the spatial lag model $\mathbf{y} = \rho \mathbf{W} \mathbf{y} + X\beta + \mu$ be estimated by OLS?



- **Spatial lag model**
  - $y = \rho W y + X\beta + \mu$ (3)

- **We cannot estimate this by OLS because of reverse causality**

- **Recall AR-model:**

  $y_t = \rho y_{t-1} + X\beta + \mu_t$ (4)
  - **We can estimate this in principle by OLS because y_{t-1} is not influenced by y_t**

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- Spatial lag model

- **Estimate with OLS?**
  - **Let's simplify** (3) **to**

    $$\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{\mu} \qquad (3')$$

- **The estimator for** ρ **yields:**

$$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{y}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

→ **Show that** $\hat{\rho}_{OLS}$ **is biased when** $\mathrm{cov}(\boldsymbol{y}, \boldsymbol{\mu}) \neq 0$

Consider estimating $\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mu$ by OLS. Show that ρ_{OLS} is biased when $\mathrm{cov}(\mathbf{y}, \mu) \neq 0$.

7

I am ready!

7

- **Spatial lag model**

- **Estimate with OLS?**
  - **Let's simplify** (3) **to**

    $\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\mu}$ (3')

- **The estimator for** ρ **yields:**

  $$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{y}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

- **If we plug-in** (3') **we get:**

  $$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'(\rho\boldsymbol{W}\boldsymbol{y}+\boldsymbol{\mu})}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

  $$\hat{\rho}_{OLS} = \rho + \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{\mu}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

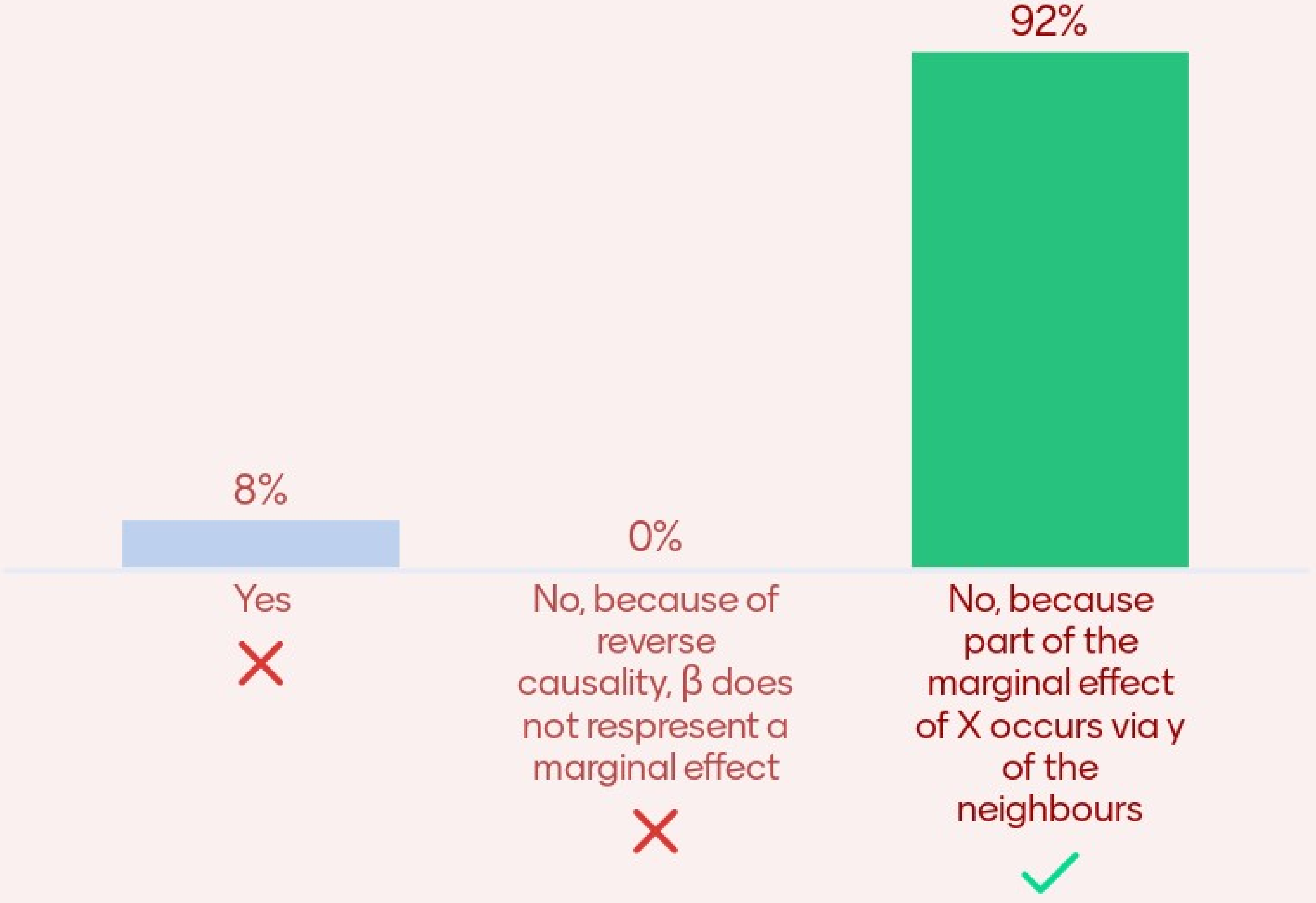
- **Hence, when** $\mathrm{cov}(\boldsymbol{y}, \boldsymbol{\mu}) \neq 0$, $\hat{\rho}_{OLS}$ **is biased**

- Spatial lag model

- Use maximum likelihood (ML) estimator
  - Selects the set of values of the model parameters that maximizes the likelihood function

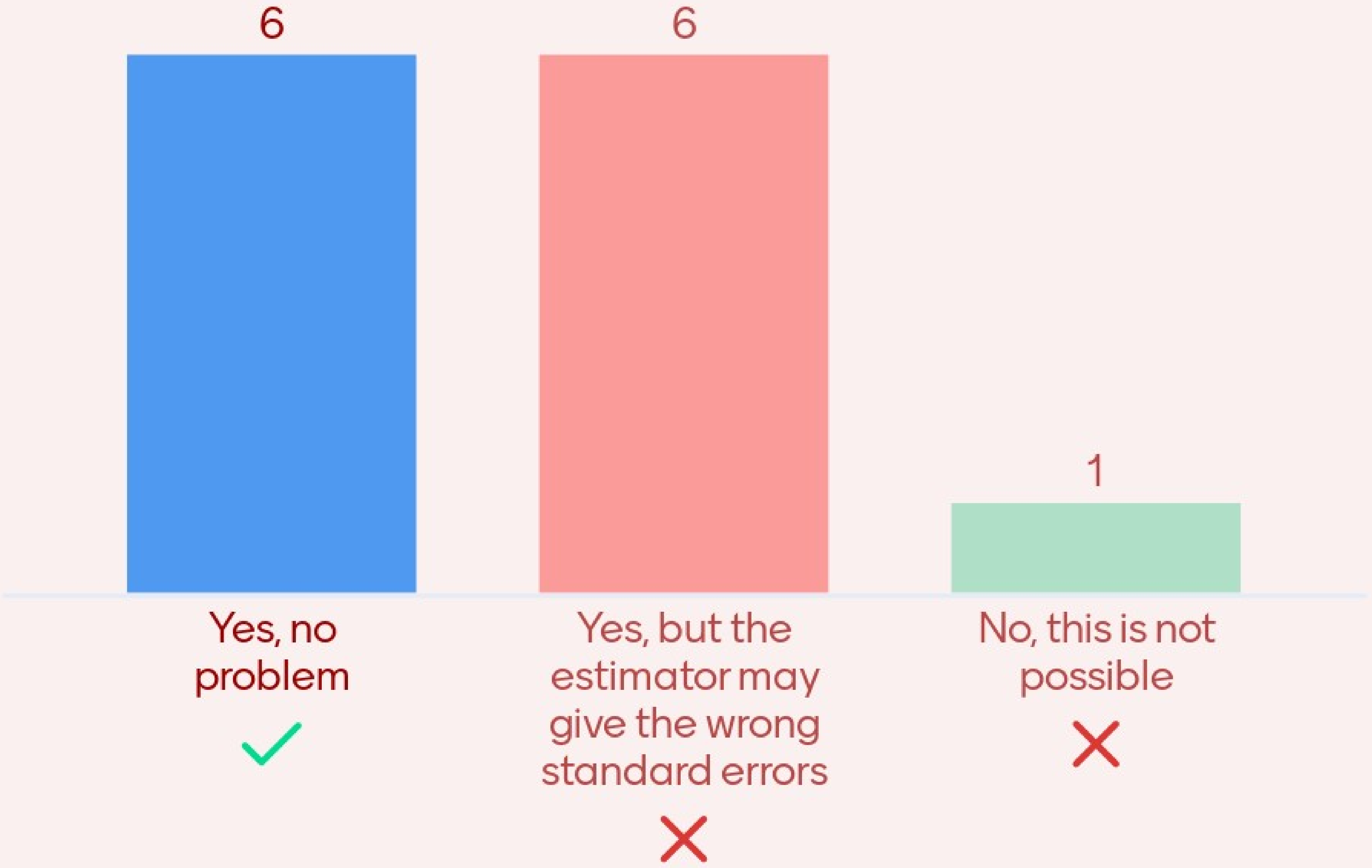
- Instrumental variables (IV)
  - Instruments for y may be WX and $W^2\mathrm{X}^2$
  - Less efficient than ML, but feasible for ‘large’ datasets
  - *e.g.* Kelejian and Prucha (1998)

Assume you use Maximum Likelihood? Does β represent a marginal effect in a spatial lag model $\mathbf{y} = \rho\mathbf{W}\mathbf{y} + X\beta + \mu$.



- **Spatial cross-regressive model**
  - $y = X\beta + \gamma WX + \mu \qquad (5)$
  - $\rho = 0, \gamma \neq 0, \lambda = 0$

Can the spatial cross-regressive model $\mathbf{y} = X\beta + \gamma\mathbf{WX} + \mu$ be estimated by OLS?



- **Spatial cross-regressive model**
  - $y = X\beta + \gamma WX + \mu$ (5)

- **Include (transformations) of exogenous variables in the regression**
  - **OLS is fine!**

- **Autocorrelation is local**

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- **Spatial error model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, with $\boldsymbol{\epsilon} = \lambda \mathbf{W}\boldsymbol{\epsilon} + \boldsymbol{\mu}$ (6)
  - $\rho = 0, \gamma = 0, \lambda \neq 0$

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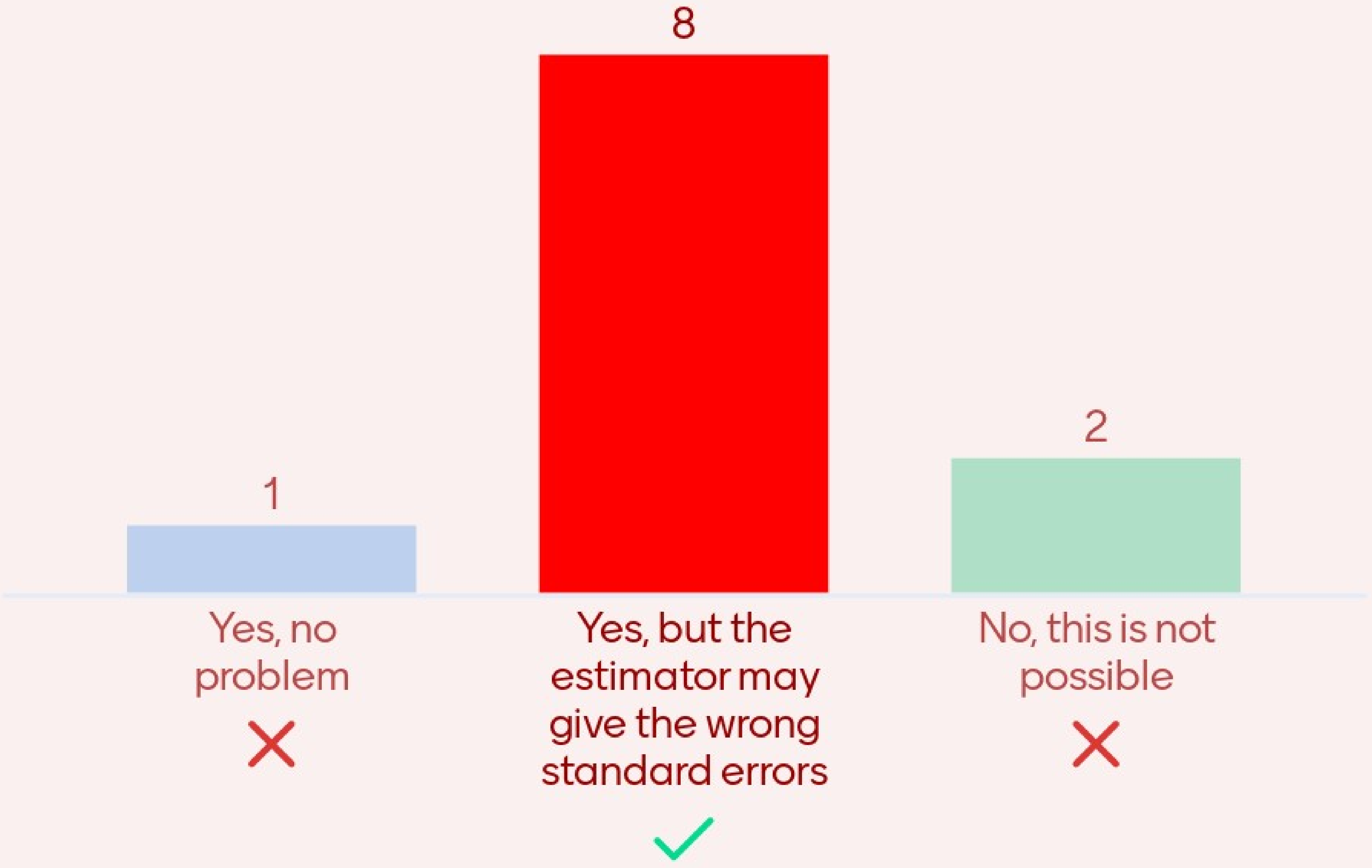
Can the spatial error model $\mathbf{y} = X\beta + \lambda \mathbf{W}\epsilon + \mu$ be estimated by OLS?



- **Spatial error model**
  - $y = X\beta + \epsilon, \text{ with } \epsilon = \lambda W\epsilon + \mu$ (6)

- **Omitted spatially correlated variables**
  - e.g. Ad-hoc defined boundaries
  - Uncorrelated to X!

- **Consistent estimation of parameters β**
- **But: inefficient**
  - ϵ are not i.i.d.
  - Standard errors are higher in OLS
  - β may be different in 'small' samples

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Spatial econometrics (3)

Applied Econometrics for Spatial Economics

Hans Koster
Professor of Urban Economics and Real Estate

2. Spatial regressions (continued)

- **Spatial lag model**
  - $y = \rho W y + X\beta + \mu$ (3)
  - $\rho \neq 0, \gamma = 0, \lambda = 0$
  - Spatial dependence in dependent variables

- **Spatial cross-regressive model**
  - $y = X\beta + \gamma W X + \mu$ (5)

- **Spatial error model**
  - $y = X\beta + \epsilon$, with $\epsilon = \lambda W\epsilon + \mu$ (6)

2. Spatial regressions (continued)

- Three issues are on the table
  1. When should you use these models?
  2. Which of the models should you choose?
  3. Can we combine these different spatial models?

1. When should you use these models?

- **Test for spatial effects**
  - H_0: No spatial dependence
- **Estimate standard OLS, $y = X\beta + \epsilon$**
  - Calculate Moran's I using $\hat{\epsilon}$
  - $I = \frac{R}{S_0} \times \frac{\hat{\epsilon}' W \hat{\epsilon}}{\hat{\epsilon}' \hat{\epsilon}}$
- **Moran's I does have a rather uninformative alternative hypothesis**
  - H_A: Spatial dependence…

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1. When should you use these models?

- **However,**
  - **Spatial errors and lags may be correlated**
  - **May also be both present**

- **Use *robust* LM tests**
  - LM_ρ^* **adds correction factor for potential spatial error**
  - LM_λ^* **adds correction factor for potential spatial lag**
  - **Complex formulae!**

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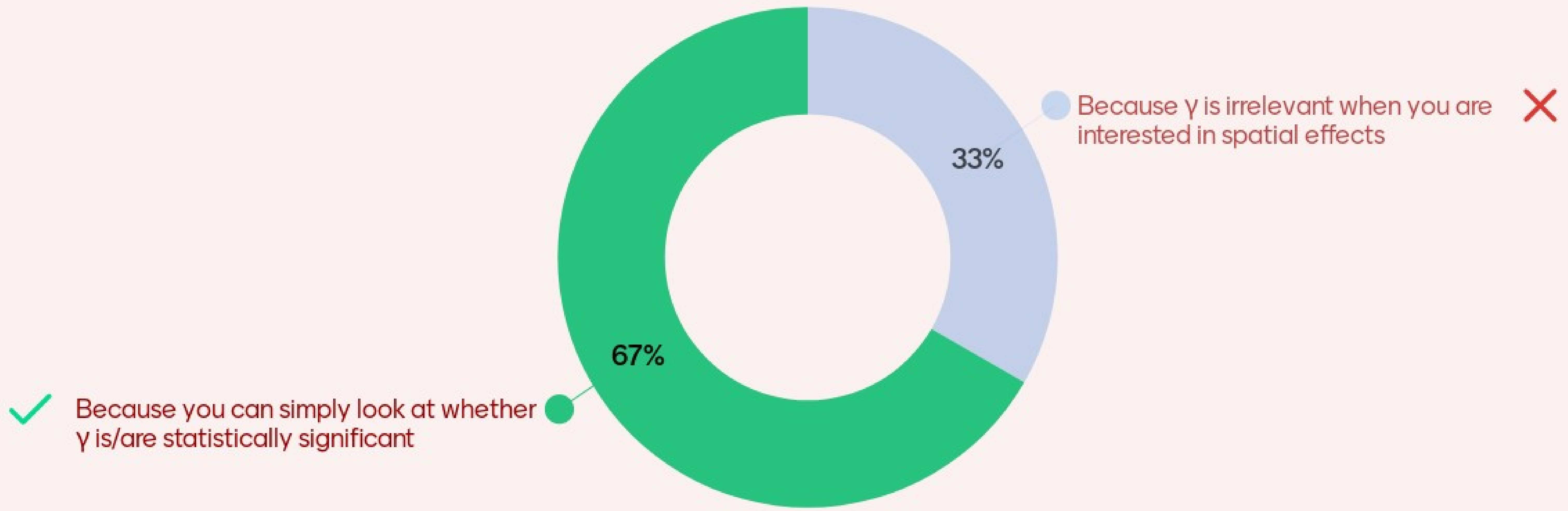
Why may we not discuss a test for the importance of spatial cross-regressive model?



3. Can we combine these spatial models?

- In practice, both a spatial lag and spatial error may be present

- How to estimate?
  - Use Kelejian and Prucha's GS2SLS method
  - Three-stage procedure!

3. Can we combine these spatial models?

- **Complicated stuff!**

- **Let software do the difficult calculations!**
  - SPAUTOREG **in STATA**
  - SPIVREG **in STATA**

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3. Mostly pointless spatial econometrics?

- Gibbons and Overman (2012)
  - *"Mostly pointless spatial econometrics?"*

- We are interested to identify causal impacts $\boldsymbol{\beta}$:
  $y = X\beta + \mu$

- Typical features of spatial data
  - Unobserved variables correlated with X
  - Omitted variable bias!
  - Large datasets

3. Mostly pointless spatial econometrics?

- **Tempting to 'fix' omitted variable bias by including a spatial lag**

- **Let's consider again:**
  $\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}$

- **Reduced-form:**
  $\boldsymbol{y} = \rho \boldsymbol{W}(\rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}) + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}$
  $\boldsymbol{y} = \rho \boldsymbol{W}(\rho \boldsymbol{W}(\rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}) + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}) + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}$
  …
  $\boldsymbol{y} = \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{W} \boldsymbol{X} \rho + \boldsymbol{W}^2 \boldsymbol{X} \rho^2 + \boldsymbol{W}^3 \boldsymbol{X} \rho^3 + [\dots] + \tilde{\boldsymbol{\mu}}$

… The last equation suggests that in the end y is just a non-linear function of the X-variables

3. Mostly pointless spatial econometrics?

- Reduced-form of spatial lag model ≈ spatial cross-regressive model
  - It is hard to prove that the spatial lag model is the 'right' model
  - So, it is hard to distinguish empirically between the two types of models
  - Only when there is a structural (network) model, a spatial lag may be appropriate

3. Mostly pointless spatial econometrics?

- The spatial lag model *does not* solve the problem of omitted variable bias!
  - Think of real exogenous sources of variation in X to identify β
  - Use instruments or quasi-experiments
  - More discussion on identification strategies in last week!

- Estimate spatial error model?
  - Spatial datasets are typically large
  - Efficiency issues are *usually* not so important

When would you use spatial econometric techniques *(multiple answers can be correct)*?

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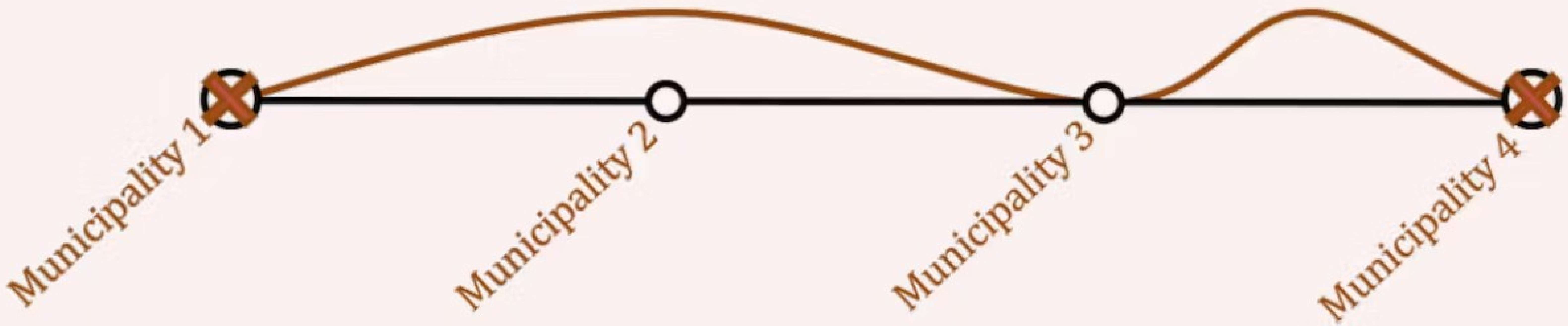
3. Mostly pointless spatial econometrics?

- Why then use spatial econometrics?
  1. Exploratory tool to investigate spatial autocorrelation
  2. Test for spatial dependence and heterogeneity, also in quasi-experiments and when using instruments
  3. Investigate whether results are robust to spatial autocorrelation (using different W)
  4. Spatial cross-regressive models are often useful and straightforward to interpret

Koster, Tabuchi & Thisse (2022, *JoEG*)

- Modern economies invest a sizable amount of money into high-speed rail
- We study the impact of high-speed rail stations on 'intermediate' places
- Local policy makers lobby for the opening of a station, but is this a good idea?

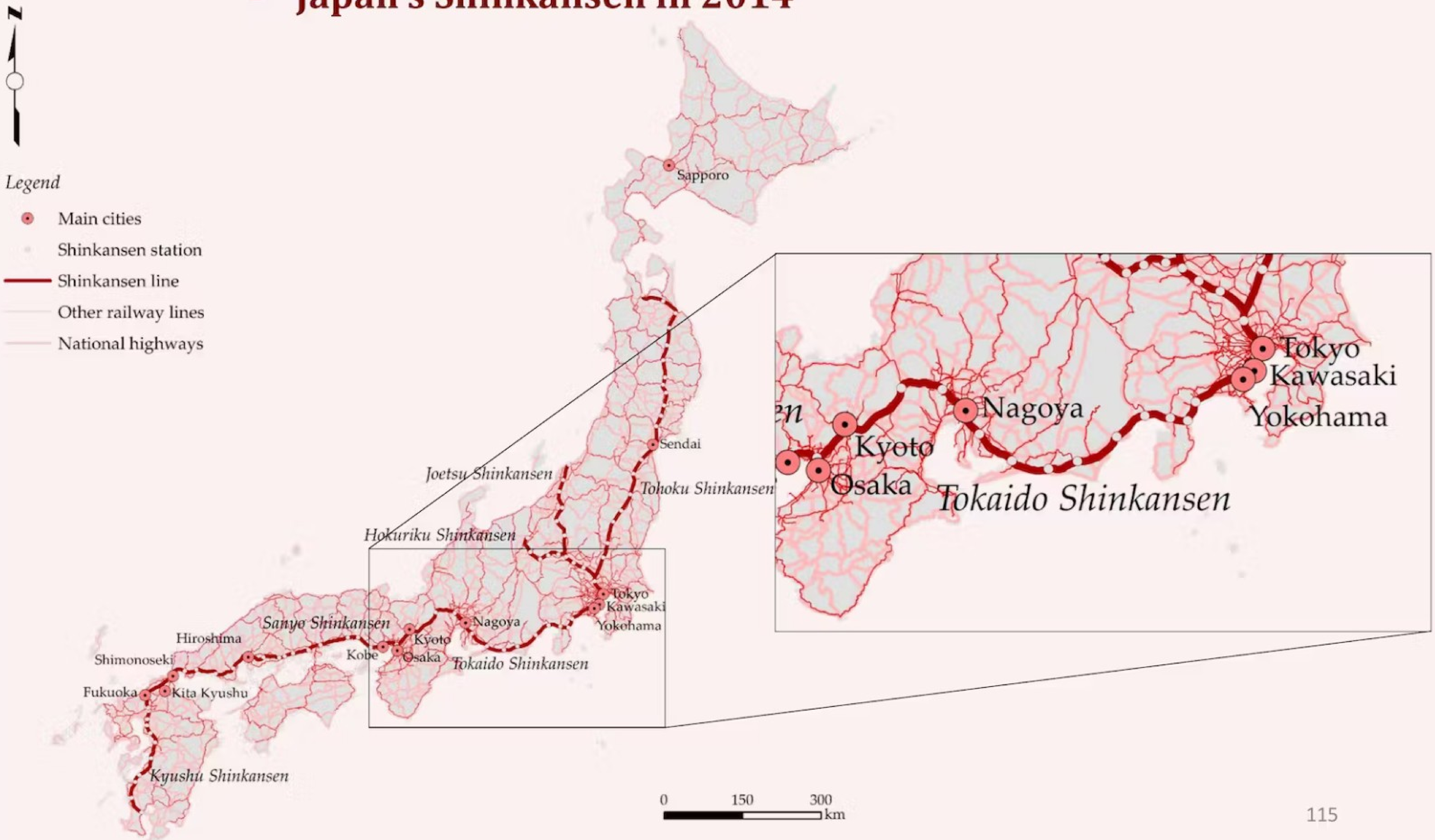


Three potential effects on employment in intermediate places:

+ A better connection reduces the need for firms to locate near large markets with high demand for goods and services
– A better connection to local markets reduces the need to locate near local markets
– When firms start to concentrate in local markets, competition becomes tougher

3. Mostly pointless spatial econometrics?

- **Japan's Shinkansen in 2014**



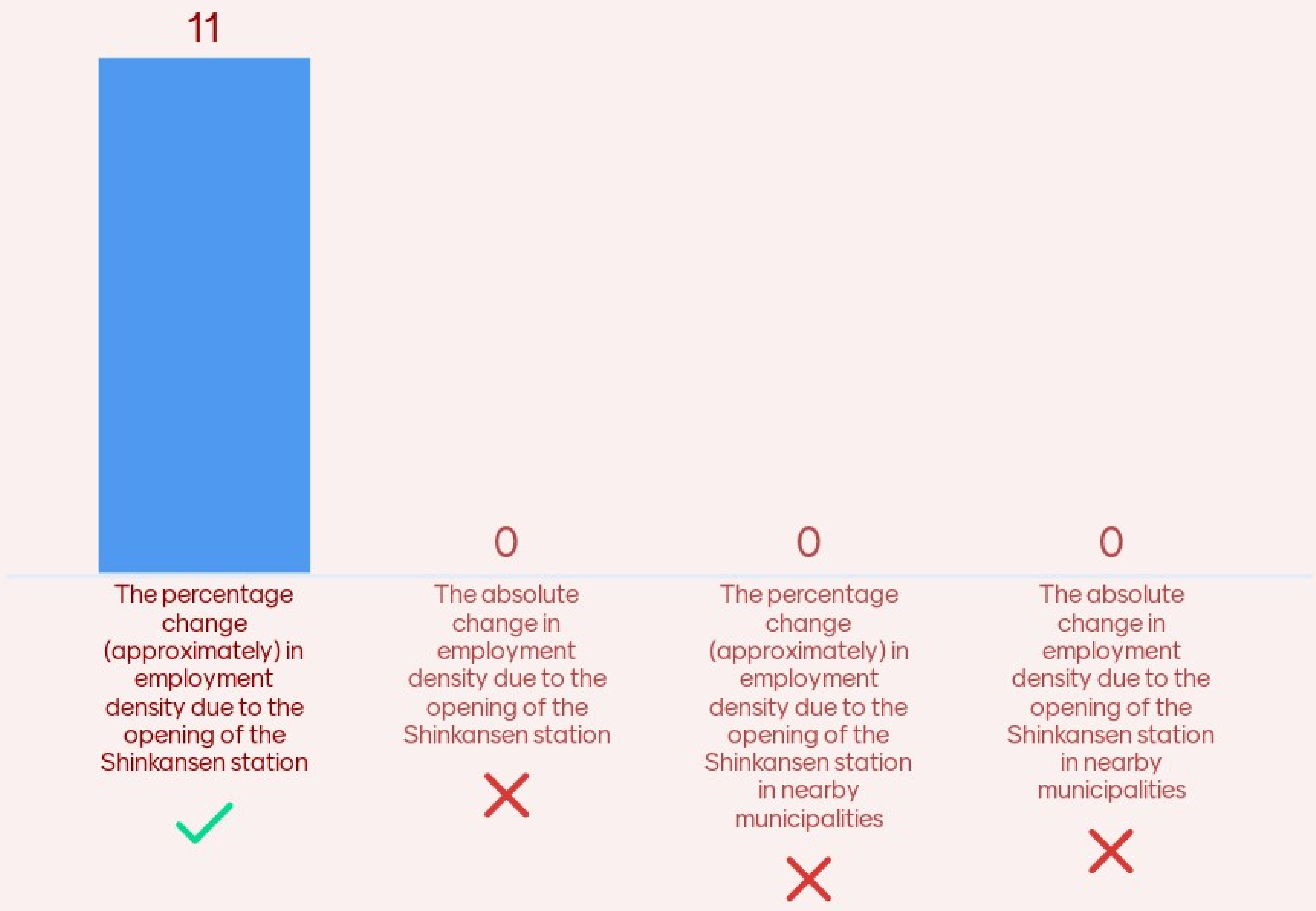
3. Mostly pointless spatial econometrics?

- **We estimate the following regression**

$$\Delta \log e = \alpha + \beta s + \boldsymbol{X}\gamma + \epsilon$$

where s captures a dummy whether a municipality has a station

With $\Delta \log e = \alpha + \beta s + \mathbf{X}\gamma + \epsilon$, what does β capture?



- **We estimate the following regression**

$$\Delta \log e = \alpha + \beta s + \boldsymbol{X}\gamma + \epsilon$$

where s captures a dummy whether a municipality has a station

- **Let us allow for spatial effects**
  - ***e.g.* because nearby stations have effects**

- **We therefore extend the baseline equation**

$$\Delta \log e = \alpha + \beta_0 s + \beta_1 \boldsymbol{W} s + \boldsymbol{X}\gamma + \epsilon$$

where $\epsilon = \lambda \boldsymbol{W} \epsilon + \mu$ and $\boldsymbol{W}$ is a row-standardised inverse-distance weight matrix

With $\Delta \log e = \alpha + \beta_0 s + \beta_1 \mathbf{W} s + \mathbf{X}\gamma + \epsilon$, what does β_1 capture?

1. Introduction
2. Spatial regressions
3. Mostly pointless?
4. Summary

Results

Table 5.1: The opening of a Shinkansen station

(Dependent variable: the log of the change in the employment density between 1957 and 2014)

	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	GS2SLS	GS2SLS	GS2SLS
	Baseline OLS	Spatial cross-regressive model	Spatial error model	Spatial lag model	All spatial effects
Shinkansen station in 2014	-0.2796**	-0.2814**	-0.2034*	-0.2167*	-0.2182*
	(0.1218)	(0.1198)	(0.1233)	(0.1246)	(0.1239)
Spatial effects:					
W· Shinkansen station in 2014		-11.1404***			-2.6923
		(2.8048)			(3.1049)
$\mathbf{W}\cdot\epsilon$			2.0174***		0.3840
			(0.3265)		(0.5581)
$\mathbf{W}\cdot\log\Delta e$				1.2501***	1.2290***
				(0.1878)	(0.2483)
Region fixed effects (8)	Yes	Yes	Yes	Yes	Yes
Number of observations	1,412	1,412	1,412	1,412	1,412
R^2	0.206	0.211			
Pseudo-R^2			0.202	0.225	0.226

Notes: **W** is a row-standardized inverse distance-weight matrix. We exclude municipalities that are centres of metropolitan or micropolitan areas. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

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Koster, Tabuchi & Thisse (2022, *JoEG*)

- The impact of a Shinkansen station reduces employment density by ≈20-25%
  - Hence, a Shinkansen station *does not benefit* intermediate places

- Spatial cross-regressive model
  - A standard deviation increase in Ws, employment density decreases by 6.8%
  - $Ws =$ the spatially weighted number of Shinkansen stations in nearby municipalities

- Spatial error and lag effects are relevant — unrealistically high spatial parameters
  - More importantly, the main effect is hardly influenced by the inclusion of spatial effects

Spatial econometrics:

- Spatial data:
  - No natural origin, reciprocity, multidirectional
  - Define spatial relationships by the spatial weight matrix

- Spatial regressions
  - Spatial lag model
  - Spatial cross-regressive model
  - Spatial error model
  - … Combine using advanced methods

- Spatial econometrics are a useful tool, *but* not a way to identify causal effects

Spatial econometrics (3)

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